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Money and credit as means of payment: A new monetarist approach ☆

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Abstract

We study the choice of payment instruments in a model with money and credit, where sellers must invest in a record-keeping technology to accept credit and buyers have limited commitment. Our model captures the two-sided market interaction between consumers and retailers that can generate multiple equilibria. Limited commitment yields an endogenous debt limit that depends on monetary policy. Money and credit coexist for a range of parameters, and bargaining related hold-up problems can lead to inefficiencies in the adoption of monitoring technologies. Changes in monetary policy generate multiplier effects in the credit market due to complementarities between consumer borrowing and the adoption of credit by merchants.

JEL classification: D82; D83; E40; E50

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1. Introduction

Much of the U.S. economy runs on debt, owed primarily to the development of credit cards that serve both as a payment instrument and a means for unsecured borrowing.[1] However, recent data suggest while consumers are increasingly relying on credit cards, they are not completely abandoning cash (Foster et al., 2011; Briglevics and Schuh, 2013). As consumers change the way they pay and businesses change the payments they accept, it is increasingly important to understand how consumer behavior affects merchant behavior and vice versa. This dynamic often generates complementarities and network externalities, a key feature of the retail payment market. These trends in the payments landscape raise many questions for monetary theory and policy. Under what conditions can money and credit coexist? Can improved access to record-keeping drive out money? And in economies where both money and credit are used, how does monetary policy affect output and welfare through the credit channel?

To address these questions, we propose a unified model of money and credit that integrates several insights from modern monetary theory, and is also analytically tractable and amenable to policy analysis. Instead of taking payment arrangements as given, our approach is in the tradition of New Monetarist Economics where the use of money and credit arise endogenously.[2] To capture the two-sided nature of actual payment systems, our model focuses on the interaction between consumers (buyers, or borrowers) and retailers (sellers, or lenders). The fundamental distinction between monetary and credit trades is that the former is settled on the spot while the latter involves delayed settlement and record-keeping to track transactions and enforce repayment. While many economies now feature the widespread adoption of both money and credit, getting money and credit to coexist in theory is a more delicate issue. A key insight from Kocherlakota (1998) is that so long as credit is feasible, there is no social role for money, and if money is valued, then credit cannot be sustained.

To generate a role for both money and credit, our model features two frictions motivating payment decisions: imperfect record-keeping and limited commitment. For credit to be feasible, we introduce a costly record-keeping technology that monitors and records transactions. As in Nosal and Rocheteau (2011), a seller who invests in this technology can accept an IOU from a buyer. Moreover, we follow a long tradition that views limited commitment as a key friction underlying credit market behavior. Lenders cannot force borrowers to repay debts, and default triggers a punishment that banishes agents from all future credit transactions (Kehoe and Levine, 1993; Alvarez and Jermann, 2000). In that case, a defaulter can only trade with money. Debt contracts are self-enforcing and the possibility of default yields an endogenous upper bound on credit use.

Our model generates three endogenous payment regimes: one with money only, one with credit only, and one with both. Both money and credit are used when some sellers (either exogenously or endogenously) accept cash and credit while others only take cash. However, we show

[1] Unsecured credit refers to loans not tied to other assets or secured by collateral. Credit card loans account for roughly half of all unsecured debt in the United States (Federal Reserve, 2005). The number of payments made by general-purpose credit cards rose from 15.2 billion to 19.0 billion between 2003 and 2006 in the U.S. (Gerdes, 2008).

[2] A discussion of the New Monetarist approach is in Williamson and Wright (2010) and a textbook treatment is in Nosal and Rocheteau (2011).

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that imperfect record-keeping is a necessary but not sufficient ingredient for money and credit to coexist. In particular, moderate inflation is a second necessary ingredient for the coexistence of money and credit. The reason is that inflation now has two effects: a higher inflation rate lowers the rate of return on money and makes default more costly. Consequently, inflation cannot be too high so that money is not valued, and inflation cannot be too low so that borrowers renege on their debts.[3] Moreover, there is a tight link between monetary policy and individual substitution patterns. By raising the cost of default and lowering the rate of return on money, higher inflation relaxes credit constraints and induces agents to shift from money to credit. Hence when both money and credit are used, an exogenous change in policy that raises debt limits will not completely crowd out money. This result holds so long as a fraction of sellers still take cash, thereby breaking the neutrality that arises in economies with perfect access to record-keeping (Gu et al., 2015).

In addition, the model highlights two important channels for monetary policy to affect equilibrium outcomes. Compared with an environment with perfect enforcement, borrowing constraints depend on monetary policy and market fundamentals. In particular, the channel through which monetary policy affects macroeconomic outcomes is through buyers' portfolio holdings, sellers' investment decisions, and the endogenous credit constraint. If sellers must invest ex-ante in a costly technology to record credit transactions, there are strategic complementarities between the seller's decision to invest and the buyer's incentive to repay. When more sellers accept credit, the gain to buyers from access to credit increases. This raises their incentives to repay, which in turn increases an individual seller's incentives to invest and hence the fraction of credit trades. This complementarity leads to feedback effects that can generate multiple steady-state equilibria. Moreover, this multiplier effect helps rationalize the rapid proliferation of unsecured credit beyond what can be explained by technological progress alone.[4]

In modern monetary theory, earlier papers studying the coexistence of money and credit include Shi (1996), Kocherlakota and Wallace (1998), and Cavalcanti and Wallace (1999). More recently, there are several models of money and credit based on the Lagos and Wright (2005) model, including Berentsen et al. (2007), Telyukova and Wright (2008), Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2013), Bethune et al. (2014), Liu et al. (2014), and Gu et al. (2015). However in all these approaches, only an exogenous subset of agents can use credit while the acceptability of credit is endogenous in this paper. Dong (2011) also models costly credit but focuses on the buyer's choice of credit. This distinction is important since heterogeneity on the buyer's side and not the seller's side implies that trades occur with money only or credit only, not with *both*.

Our model of endogenous record-keeping is similar to Nosal and Rocheteau (2011), though a key distinction is we derive an endogenous debt limit under limited commitment instead of assuming repayments are perfectly enforced.[5] Having limited commitment provides an important channel for monetary policy to affect equilibrium borrowing. As just one example, our theory implies a fall in the nominal interest rate tightens credit constraints which generates a decline in consumer borrowing, even holding fixed the aggregate acceptability of credit by merchants. Consequently, monetary policy affects credit market outcomes along two dimensions: the extensive margin (number of credit trades) and the intensive margin (amount borrowed per trade).

[3] See also Berentsen et al. (2007), Sanches and Williamson (2010), Bethune et al. (2014), and Gu et al. (2015).

[4] This channel mimics the mechanism behind two-sided markets in actual payment systems (Chakravorti, 2010).

[5] Our formalization of costly record-keeping is similar to the information acquisition decision in Nosal and Rocheteau (2011) and Lester et al. (2012) where sellers must incur a fixed cost to authenticate and hence accept an asset for trade.

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In a related setting but with complete access to record-keeping by sellers, Gu et al. (2015) show that money is crowded out one for one when credit is also used. Despite having similar topics and methods, our paper and theirs feature different environments and we highlight different findings. As we show, imperfect record keeping is critical for obtaining the coexistence of money and credit at the individual transaction level, a role for monetary policy to affect debt limits, and strategic complementarities between equilibrium borrowing and the aggregate availability of credit. Indeed, our model nests a version of the full record-keeping model of Gu et al. (2015), in which case we obtain their neutrality result. See also Araujo and Hu (2014) who show money and credit can be coessential by having two rounds of decentralized market trades. Our analysis focuses instead on the *coexistence*, rather than coessentiality, of money and credit.

The paper proceeds as follows. Section 2 presents the benchmark environment with limited enforcement. Section 3 determines equilibrium when an exogenous fraction of sellers accept credit, and Section 4 characterizes payment regimes. In Section 5, we endogenize the acceptability of credit. Section 6 discusses welfare and investigates the welfare effects of inflation. Finally, Section 7 closes with concluding remarks.

2. Environment

Time is discrete and continues forever. There is a continuum $[0, 2]$ of infinitely lived agents, evenly divided between *buyers* and *sellers*. Each period is divided into two sub-periods where economic activity will differ. In the first sub-period, agents meet pairwise and at random in a decentralized market, called the *DM*. Sellers can produce output, $q \in \mathbb{R}+$, but do not want to consume, while buyers want to consume but cannot produce. Agents' identities as buyers or sellers are permanent. In the second sub-period, trade occurs in a frictionless centralized market, called the *CM*, where all agents can consume a numéraire good, x, by supplying labor, y, using a linear technology $f(y) = y$.

Instantaneous utility functions for buyer and sellers are separable between sub-periods and linear in the CM:

$$\mathcal{U}^b(q, x, y) = u(q) + x - y,$$
$$\mathcal{U}^s(q, x, y) = -c(q) + x - y.$$

We assume u and c are C^2 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $u'(0) = \infty$. Also, let $q^* = \arg\max_q [u(q) - c(q)] > 0$. All goods are perishable and agents' discount factor between periods is $\beta = \frac{1}{1+r} \in (0, 1)$.

The only asset in the economy is fiat money, which is perfectly divisible, storable, and recognizable. Money $m_t \in \mathbb{R}_+$ is valued at ϕ_t, the price of money in terms of numéraire. Its aggregate stock, M_t, can grow or shrink each period at a constant gross rate $\gamma \equiv \frac{M_{t+1}}{M_t}$. Changes in the money supply are implemented through lump-sum transfers or taxes to buyers in the CM. We assume the government has enough enforcement in the CM so that agents will pay the lump-sum tax.[6]

[6] Alternatively, Andolfatto (2013) considers an environment where the government's enforcement power is limited and the payment of lump-sum taxes is voluntary. Punishment for failing to pay taxes in the CM is then permanent exclusion from DM trades.

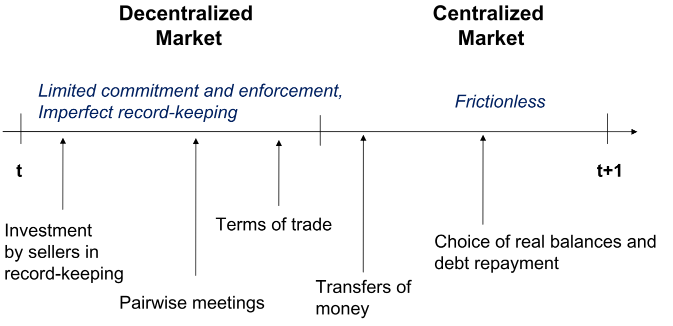

Fig. 1. Timing of a period.

There is a costly record-keeping technology that can record transactions and enforce repayment. At the beginning of the DM, sellers can invest to access this technology by paying a fixed flow cost, $\kappa \geq 0$, in units of utility. To simplify exposition, assume for now that investment in this technology is costless for a constant fraction $\Lambda \in [0, 1]$ of sellers and prohibitively costly for the remaining $1 - \Lambda$ sellers. This implies the fraction of sellers with record-keeping is Λ. In Section 5, we endogenize Λ by considering a more general cost function where individual sellers have heterogeneous costs of investing.

Buyers can issue $b \in \mathbb{R}_+$ units of one-period IOUs normalized to be worth one unit of numéraire. Loans made in the DM can be repaid in the subsequent CM. Here, sellers with access to record-keeping can make loans directly to the buyer without interacting with an intermediary, such as a bank or credit card issuer. Hence, the seller and credit card issuer is modeled as a consolidated entity. Since agents lack commitment, potential borrowers must be punished if they do not repay. We assume any default is publicly recorded by the record-keeping technology and triggers punishment that leads to permanent exclusion from the credit system. In that case, a borrower who defaults can only use money for all future transactions.[7]

Fig. 1 summarizes the timing of a period. At the beginning of the DM, a buyer matches with a seller with probability σ, where the buyer has b units of debt and m units of money, or equivalently, $z \equiv \phi m \in \mathbb{R}+$ units of real balances. Terms of trade are determined using Kalai's (1977) proportional bargaining solution. In the CM, buyers produce the numéraire good, redeem their loan, and acquire money, while sellers purchase the numéraire and get repaid.

3. Equilibrium

We focus on stationary equilibria where $\gamma \geq \beta$ and real balances are constant over time. Consequently, the rate of return on money is constant and equal to $\gamma^{-1} = \frac{\phi'}{\phi}$. In what follows, variables with a prime denote next period's variables.

3.1. Centralized market value functions

In the beginning of the CM, agents consume the numéraire good, supply labor, and readjust their portfolios. Let $W^b(z, -b)$ denote the value function of a buyer who holds $z \equiv \phi m$ units of

[7] The record-keeping technology detects default with probability one. Introducing imperfect monitoring where default is only detected probabilistically would all else equal decrease the cost of default and hence tighten credit limits. See also Bethune et al. (2014) and Gu et al. (2015).

real balances and has issued b units of debt in the previous DM. Similarly, $\widetilde{W}^b(z)$ denotes the value function of a buyer with a recorded history of default and hence does not have access to credit.

The maximization problem for a buyer who enters the CM with portfolio $(z, -b)$ is

$$W^b(z, -b) = \max_{x,y,z' \geq 0} \left\{ x - y + \beta V^b(z') \right\} \tag{1}$$

$$\text{s.t. } x + b + \phi m' = y + z + T, \tag{2}$$

where $V^b(z')$ is the buyer's continuation value in the next DM and $T \equiv (\gamma - 1)\phi M$ is the lump-sum transfer of money from the government (in units of numéraire). According to (2), the buyer finances his net consumption of the numéraire $(x - y)$, repayment of debt (b), and next period's real balances $(\phi m')$ with his current real balances $(\phi m = z)$ and the lump-sum transfer (T). Substituting $m' = z'/\phi'$ into (2) and substituting y into (1) yields

$$W^b(z, -b) = z - b + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta V^b(z') \right\}. \tag{3}$$

The buyer's lifetime utility in the CM is the sum of his real balances net of any debt to be repaid, the lump-sum transfer from the government, and his continuation value at the beginning of the next DM net of the investment in real balances. Notice W^b is linear in the buyer's total wealth, $z - b$: $W^b(z, -b) = z - b + W^b(0, 0)$. In addition, the buyer's choice of real balances next period, z', is independent of his current real balances, z.

Similarly, the value function of a buyer who defaults is

$$\widetilde{W}^b(z) = z + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta \widetilde{V}^b(z') \right\},$$

where $\widetilde{V}^b(z')$ is the continuation value of a buyer who defaults and hence loses access to credit in the next DM.

Since sellers have no strict incentive to accumulate real balances in the DM, their CM value function is

$$W^s(z, b) = z + b + \beta V^s(0),$$

where $V^s(0)$ is the seller's value function at the beginning of the following DM.

3.2. Terms of trade

Terms of trade in the DM are determined by Kalai's bargaining solution where the buyer receives a share $\theta \in (0, 1)$ of the total surplus. Upon being matched, the buyer proposes to the seller a contract (q, d, b), where q is the quantity of output the seller produces for the buyer, and (d, b) are the transfers of real balances and IOUs, respectively, from the buyer to seller.[8]

[8] Proportional bargaining has several desirable features that cannot be guaranteed with Nash bargaining, as discussed in Aruoba et al. (2007). First, it guarantees the concavity of agents' value functions. Second, the proportional solution is monotonic and hence does not suffer from a shortcoming of Nash bargaining that an agent can end up with a lower individual surplus even if the size of the total surplus increases. While some of these technical complications can be dealt with, as in Lagos and Rocheteau (2008) and Lagos (2010), or avoided altogether by giving all the surplus to the buyer, here we want to give positive surplus to the seller since later on we endogenize the acceptability of credit and need to provide sellers some incentive to invest in the record-keeping technology. Moreover, there are also normative reasons to adopt Kalai bargaining, as discussed by Bethune et al. (2015).

In general, the terms of trade only depend on the total portfolio of the buyer and the payment accepted by the seller. Consider first a match where the seller accepts credit. Notice that the surplus of a buyer who gets q for payment $d+b$ to the seller is $u(q)-d-b$, by the linearity of W^b. Similarly, the surplus of the seller is $-c(q)+d+b$. The bargaining problem simplifies to

$$(q,d,b)=\arg\max_{q,d,b}\{u(q)-d-b\} \tag{4}$$

$$\text{s.t.} \; -c(q)+d+b=\frac{1-\theta}{\theta}\left[u(q)-d-b\right], \tag{5}$$

$$d\in[0,z], b\in[0,\bar{b}], \tag{6}$$

where $d\in[0,z]$ is a feasibility constraint on the amount the buyer can transfer to the seller, while $b\in[0,\bar{b}]$ is the buyer's incentive constraint that motivates debt repayment. The threshold $\bar{b}$ is an equilibrium object and represents the endogenous borrowing limit faced by the buyer, which is taken as given in the bargaining problem. From (6), the buyer's payment constraint is:

$$d+b\le z+\bar{b}, \tag{7}$$

The bargaining solution depends on whether or not the payment constraint binds. If (7) does not bind, then the buyer has sufficient wealth to purchase the first best level of output, q^*. In that case, payment to the seller will be

$$d+b=(1-\theta)u(q^*)+\theta c(q^*).$$

If (7) binds, the buyer does not have enough payment capacity and will borrow up to his credit limit and pay the rest with any cash on hand:

$$z+\bar{b}=(1-\theta)u(q^c)+\theta c(q^c), \tag{8}$$

where $q^c\equiv q(z+\bar{b})<q^*$.

If the seller does not have access to record-keeping, credit cannot be used. In that case, the bargaining problem is described by (4)–(6) but with $b=\bar{b}=0$. If $z\ge(1-\theta)u(q^*)+\theta c(q^*)$, the buyer is unconstrained and has enough liquid wealth to purchase q^*. Otherwise, the buyer just hands over his real balances,

$$z=(1-\theta)u(q)+\theta c(q), \tag{9}$$

where $q\equiv q(z)<q^*$. The bargaining solution if the buyer defaults is similar to (9), where q is replaced with $\widetilde{q}$, and the real balances of a buyer who defaults is $\widetilde{z}=(1-\theta)u(\widetilde{q})+\theta c(\widetilde{q})$ if $\widetilde{z}<(1-\theta)u(q^*)+\theta c(q^*)$ and $\widetilde{z}=(1-\theta)u(q^*)+\theta c(q^*)$ otherwise.

3.3. Decentralized market value functions

We next describe agents' value functions in the DM. After simplification, the expected lifetime utility of a buyer with no history of default who holds z units of real balances at the beginning of the period is:

$$V^b(z)=\sigma\theta(1-\Lambda)[u(q)-c(q)]+\sigma\theta\Lambda\left[u\left(q^c\right)-c\left(q^c\right)\right]+z+W^b(0,0), \tag{10}$$

where we have used the bargaining solution and the fact that the buyer will never accumulate more real balances than he would spend in the DM. According to (10), a buyer in the DM is matched with a seller who does not have access to record-keeping with probability $\sigma(1-\Lambda)$, receives θ of the match surplus, $u(q) - c(q)$, and can only pay with money. With probability $\sigma\Lambda$, a buyer matches with a seller with access to record-keeping, in which case he gets θ of the match surplus, $u(q^c) - c(q^c)$, and can pay with both money and credit. The last two terms result from the linearity of W^b and is the value of proceeding to the CM with one's portfolio intact.

Similarly, the expected lifetime utility of a buyer who defaults is

$$\widetilde{V}^b(z) = \sigma\theta[u(\widetilde{q}) - c(\widetilde{q})] + z + \widetilde{W}^b(0),$$

which reflects the fact that a buyer who defaults loses access to credit and can only consume $\widetilde{q}$ by trading with money. However if money is not valued, then a buyer who defaults cannot trade at all in the DM.

We now turn to the buyer's choice of real balances. Substituting $V^b(z)$ from (10) into (3), the choice of real balances for a buyer with access to credit solves

$$\max_{z \geqslant 0} \left\{-iz + \sigma(1-\Lambda)\theta S(z) + \sigma\Lambda\theta S(z+\bar{b})\right\}, \tag{11}$$

where $S(\cdot) \equiv u[q(\cdot)] - c[q(\cdot)]$ is the total trade surplus in a bilateral match and $i \equiv \frac{\gamma-\beta}{\beta}$ is the nominal interest rate on an illiquid bond and represents the cost of holding real balances. According to (11), the buyer's choice of real balances maximizes his expected surplus in the DM net of the cost of holding money. Since the objective function (11) is continuous and maximizes over a compact set, a solution exists. In Appendix A, we show that (11) is a concave problem. Accordingly, the buyer's choice of real balances, $z \geq 0$, solves

$$-i + \sigma\theta(1-\Lambda)S'(z) + \sigma\theta\Lambda S'(z+\bar{b}) \leq 0, \tag{12}$$

where (12) is satisfied with equality if $z > 0$ and $S'(\cdot) = \frac{u'[q(\cdot)]-c'[q(\cdot)]}{\theta c'[q(\cdot)]+(1-\theta)u'[q(\cdot)]}$ represents the liquidity premium. Using a similar line of reasoning, the choice of real balances for a buyer who defaults and hence loses access to credit, $\widetilde{z} \geq 0$, solves

$$-i + \sigma\theta S'(\widetilde{z}) \leq 0, \tag{13}$$

where (13) is satisfied with equality if $\widetilde{z} > 0$.

Notice when all sellers have access to record-keeping ($\Lambda = 1$) and money is valued, (12) and (13) at equality implies a buyer with access to credit chooses his real balances so that his total wealth is the same as a buyer who defaults, or $z + \bar{b} = \widetilde{z}$. Intuitively, a buyer who repays his debt faces the same tradeoff at the margin as a buyer who defaults. Moreover, when $\Lambda = 1$, any change in the debt limit (either exogenous or endogenous) is offset by a change of the same magnitude in real balances. To see this when $\bar{b}$ is exogenous, differentiate (12) to obtain

$$\frac{dz}{d\bar{b}} = -\left[1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''(z+\bar{b})}\right]^{-1} \in [-1, 0).$$

When $\Lambda = 1$, $\frac{dz}{d\bar{b}} = -1$, in which case any increase in the debt limit leaves the buyer's total wealth unchanged. Importantly, this result does not hold more generally for our model with $\Lambda \in (0, 1)$ due to the subset of trades that do not require credit. We give a more formal treatment of this outcome in Section 4.3.1 when characterizing an equilibrium with both money and credit.

Finally, notice under perfect enforcement, buyers are never constrained by the limit $b \leq \bar{b}$ in the fraction Λ of meetings, and can borrow as much as they want to finance the first best, q^*. However, in the fraction $(1-\Lambda)$ of meetings, only money is accepted. Therefore, when money is valued, the third term on the left side of (12) goes to zero since $u'(q^*) = c'(q^*)$. In that case, the second term on the left side of (12) is increasing with Λ so that an increase in Λ decreases output, $q(z)$, and hence the buyer's real balances choice, z.

4. Endogenous credit limits

When the government's ability to enforce repayment is limited, borrowers may have an incentive to renege on their debt obligations. To support trade in a credit economy, we assume punishment for default entails permanent exclusion from the credit system. In that case, a borrower who defaults can only use money for all future transactions.

The equilibrium credit limit, $\bar{b}$, is determined so the buyer voluntarily repays his debt. The buyer's incentive compatibility constraint for voluntary debt repayment is

$$W^b(z, -b) \geqslant \widetilde{W}^b(z),$$

where $W^b(z, -b)$ is the value function of a buyer who repays his debt at the beginning of the CM, and $\widetilde{W}^b(z)$ is the value function of a buyer who defaults. Hence, a buyer chooses to repay his debt if his lifetime utility from repaying is higher than his lifetime utility from reneging, or

$$-b + W^b(z, 0) \geq \widetilde{W}^b(z). \tag{14}$$

According to (14), a buyer who honors his debt obligation repays his debt, $-b$, and enters the CM with z units of real balances and access to future credit, while a buyer who defaults enters the CM with a history of recorded default and hence loses access to future credit. By the linearity of W^b and $\widetilde{W}^b$, (14) can be rewritten as

$$b \leq \bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0), \tag{15}$$

where $\bar{b}$ is the endogenous upper bound on credit. According to (15), the amount borrowed can be no larger than the expected cost of default, which is the difference between the lifetime utility of a buyer with access to credit and the lifetime utility of a buyer permanently excluded from using credit. Due to absence of wealth effects, the debt limit $\bar{b}$ is independent of the buyer's real balances held in the beginning of the CM.

Lemma 1. *The equilibrium debt limit, $\bar{b}$, solves*

$$\begin{aligned} r\bar{b} &= \max_{z \geq 0} \left\{-iz + \sigma\theta \left[(1-\Lambda) S(z) + \Lambda S\left(z + \bar{b}\right)\right]\right\} - \max_{\tilde{z} \geq 0} \{-i\tilde{z} + \sigma\theta S(\tilde{z})\} \\ &\equiv \Omega(\bar{b}). \end{aligned} \tag{16}$$

The left side of (16), $r\bar{b}$, represents the return from borrowing a loan of size $\bar{b}$. The right side, $\Omega(\bar{b})$, is the flow cost of default, which equals the surplus from not having access to credit. According to (16), a solution to the debt limit is a fixed point of $r\bar{b} = \Omega(\bar{b})$.

Definition 1. Given Λ, a stationary equilibrium is a list $(q, q^c, z, \tilde{z}, \bar{b})$ that solves (8), (9), (12), (13), and (16). Equilibrium is said to be *monetary* if $\phi > 0$ and *non-monetary* if $\phi = 0$.

To characterize an equilibrium with credit, we start by establishing some key properties of the flow cost of default, $\Omega(\bar{b})$. Notice that the value of $\Omega(\bar{b})$ depends on the buyer's off-equilibrium choice of real balances in the event of default, $\tilde{z}$, which itself depends on the value of money, ϕ.

We therefore distinguish two cases for the flow cost of default: (*1*) when money is not valued ($\phi = 0$) and (2) when money is valued ($\phi > 0$).

When money is not valued and hence $\phi m = z = 0$, a buyer rationally anticipates that punishment for default is permanent autarky. Since defaulters will not be able to trade in the future, $\widetilde{z} = 0$ in steady state. Accordingly, the right side of (16) when money is not valued is given by

$$\Omega(\bar{b})|_{\phi=0} \equiv \sigma\Lambda\theta S(\bar{b}).$$

When money is valued and $\phi m = z > 0$, a buyer anticipates being able to trade with money in the future. In turn, $\widetilde{z} > 0$ solves (13) at equality, and the flow cost of default is given by

$$\Omega(\bar{b})|_{\phi>0} \equiv \max_{z>0} \left\{-iz + \sigma\theta\left[(1-\Lambda)S(z) + \Lambda S\left(z + \bar{b}\right)\right]\right\} - \max_{\widetilde{z}>0} \{-i\widetilde{z} + \sigma\theta S(\widetilde{z})\}.$$

To describe the properties of $\Omega(\bar{b})|_{\phi=0}$ and $\Omega(\bar{b})|_{\phi>0}$, it is useful to define a critical value for the debt limit. Consider a meeting where the seller has access to record-keeping. Suppose $i > 0$ and define $\bar{b}_1$ as the threshold for the debt limit, above which agents can finance the first-best with credit. From the bargaining solution, the buyer has sufficient wealth to obtain q^* when $(1-\theta)u(q^*) + \theta c(q^*) \le z + \bar{b}$. If money is not valued, then $\bar{b}_1 = (1-\theta)u(q^*) + \theta c(q^*)$. If money is valued, then (12) is satisfied with equality, and the threshold for the debt limit, above which buyers can obtain the first-best is given by $\bar{b}_1 - \bar{z}$ where $\bar{z}$ solves

$$i = \sigma\theta(1-\Lambda)S'(\bar{z}).$$

In that case, $q^c = q^*$ if $\bar{b} \ge \bar{b}_1 - \bar{z}$ and $q^c < q^*$ otherwise.

We now summarize some key properties of $\Omega(\bar{b})|_{\phi=0}$ and $\Omega(\bar{b})|_{\phi>0}$ in the lemma below. In Section 4.3, we consider a special case of the model with perfect record-keeping ($\Lambda = 1$), which has some qualitatively different properties from the model with imperfect record-keeping studied below.

Lemma 2. *Given $\Lambda \in (0, 1)$ and $i > 0$, the correspondence $\Omega(\bar{b})$ has the following properties.*

1. *When money is not valued, $\Omega(\bar{b})|_{\phi=0}$ is such that*
   (a) $\Omega(0)|_{\phi=0} = 0$,
   (b) $\Omega'(0)|_{\phi=0} = \frac{\sigma\Lambda\theta}{1-\theta} > 0$,
   (c) $\Omega'(\bar{b})|_{\phi=0} = \begin{cases} \sigma\theta\Lambda S'(\bar{b}) > 0 & \text{if } \bar{b} < \bar{b}_1 \\ 0 & \text{if } \bar{b} \ge \bar{b}_1, \end{cases}$
   (d) *$\Omega(\bar{b})|_{\phi=0}$ is strictly increasing and concave for all $\bar{b} < \bar{b}_1$ and is constant at $\sigma\theta\Lambda S^*$ where $S^* \equiv u(q^*) - c(q^*)$ otherwise.*
2. *When money is valued, $\Omega(\bar{b})|_{\phi>0}$ is such that*
   (a) $\Omega(0)|_{\phi>0} = 0$,
   (b) $\Omega'(0)|_{\phi>0} = i\Lambda > 0$,
   (c) $\Omega'(\bar{b})|_{\phi>0} = \begin{cases} \sigma\theta\Lambda S'(z+\bar{b}) > 0 & \text{if } \bar{b} < \bar{b}_1 - \bar{z} \\ 0 & \text{if } \bar{b} \ge \bar{b}_1 - \bar{z}, \end{cases}$
   (d) *$\Omega(\bar{b})|_{\phi>0}$ is strictly increasing and concave for all $\bar{b} < \bar{b}_1 - \bar{z}$ and is constant at $\sigma\theta\Lambda S^*$ otherwise.*

Fig. 2 illustrates the key properties of the flow cost of default from Lemma 2. The top curve is $\Omega(\bar{b})|_{\phi=0}$ while the bottom curve is $\Omega(\bar{b})|_{\phi>0}$. With Kalai bargaining, both curves are continuous, increasing, and concave. Consider first $\Omega(\bar{b})|_{\phi=0}$ where money is not valued. When $\bar{b} < \bar{b}_1$,

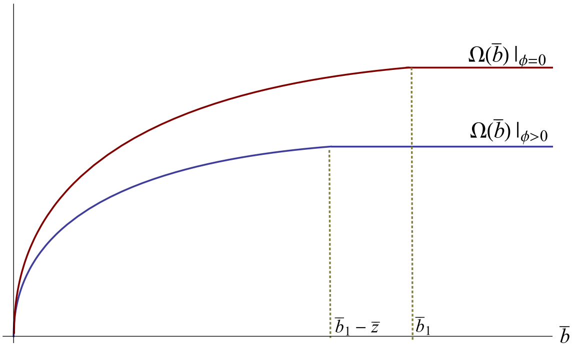

Fig. 2. Flow cost of default, $\Omega(\bar{b})$, when $\Lambda \in (0, 1)$.

credit is tight and the flow cost of default increases with the size of the loan. Since a higher credit line makes default more tempting, a harsher punishment is needed to ensure credit is incentive feasible. When $\bar{b} \geq \bar{b}_1$, credit alone is sufficient to finance the first best, and the flow cost of default becomes constant and equal to $\sigma\theta\Lambda S^*$. Similar explanations apply for $\Omega(\bar{b})|_{\phi>0}$, except the threshold for the debt limit above which the first-best can be attained is less stringent and given by $\bar{b}_1 - \bar{z}$.

From Lemma 2, notice when $i < \frac{\sigma\theta}{1-\theta}$, the slope of $\Omega(\bar{b})|_{\phi>0}$ at $\bar{b} = 0$, which is $\sigma\theta\Lambda S'(z) = i\Lambda$, is strictly less than the slope of $\Omega(\bar{b})|_{\phi=0}$ at $\bar{b} = 0$, which is $\sigma\theta\Lambda S'(0) = \frac{\sigma\Lambda\theta}{1-\theta}$. Moreover, $\Omega'(\bar{b})|_{\phi>0} < \Omega'(\bar{b})|_{\phi=0}$ for all $\bar{b}$ whenever the credit constraint binds. Hence in Fig. 2, the curve $\Omega(\bar{b})|_{\phi>0}$ lies strictly below the curve $\Omega(\bar{b})|_{\phi=0}$. Intuitively, default is less costly at the margin when money is valued than when money is not valued since in the former, the buyer can still use money for future transactions.

We now turn to characterizing three types of steady-state equilibria that can arise in the model: *(i)* a pure monetary equilibrium, *(ii)* a pure credit equilibrium, and *(iii)* a money-credit equilibrium.

4.1. Pure monetary equilibrium

We start by considering a benchmark pure monetary equilibrium where money is valued and credit is not used. This type of outcome arises in environments with limited commitment and no access to record-keeping ($\Lambda = 0$), as in standard New Monetarist models. This outcome also arises in our environment for any $\Lambda \in [0, 1]$, so long as inflation is not too high.

Money is valued given there is no credit if and only if

$$i < \frac{\sigma\theta}{1-\theta} \equiv \bar{i},$$

where we have used the fact that $S(z)$ is a concave function and $S'(0) = \frac{1}{1-\theta}$. Since an equilibrium without credit always exists and is self fulfilling, a necessary and sufficient condition for a pure monetary equilibrium is $i < \bar{i}$, which is independent of Λ.

4.2. Pure credit equilibrium

In a pure credit equilibrium, money is not valued while credit is used. When money is not valued, the debt limit $\bar{b}$ solves

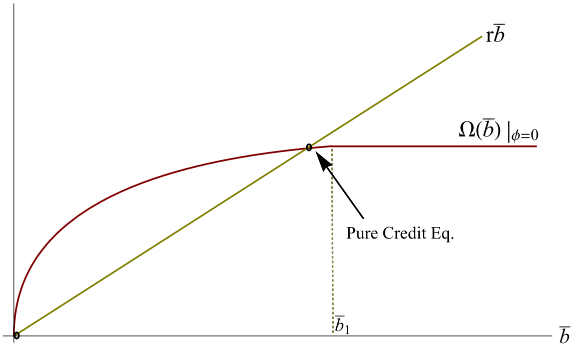

Fig. 3. Pure credit equilibrium when $\Lambda > \overline{\Lambda}$.

$$r\bar{b} = \sigma\theta\Lambda S\left(\bar{b}\right) \equiv \Omega\left(\bar{b}\right)|_{\phi=0}. \tag{17}$$

A necessary condition for credit is that the slope of $r\bar{b}$ is less than the slope of $\Omega\left(\bar{b}\right)|_{\phi=0}$ at $\bar{b} = 0$, or

$$r < \frac{\sigma\Lambda\theta}{1-\theta}, \tag{18}$$

which says that agents have to be patient enough to sustain voluntary debt repayment. Hence, when the fraction of sellers accepting credit is exogenous, there exists a threshold for the fraction of credit trades, below which $\bar{b} = 0$. From (18), credit is feasible given money is not valued if

$$\Lambda > \frac{r(1-\theta)}{\sigma\theta} \equiv \overline{\Lambda}.$$

Since there always exists an equilibrium where money is not valued, a necessary and sufficient condition for a pure credit equilibrium is $\Lambda > \overline{\Lambda}$.

Fig. 3 shows credit is feasible if $\Lambda > \overline{\Lambda}$ so that $\Omega\left(\bar{b}\right)|_{\phi=0}$ intersects with $r\bar{b}$ from above. In addition, the line $r\bar{b}$ rotates to the right as agents become more patient, which relaxes the debt limit. Even when enforcement is limited, credit alone can sustain the first best as long as agents care enough about the future, that is if

$$r \leq \frac{\sigma\Lambda\theta[u(q^*) - c(q^*)]}{(1-\theta)u(q^*) + \theta c(q^*)} \equiv r^*.$$

In contrast, credit is not incentive feasible when $\Lambda \leq \overline{\Lambda}$ as in Fig. 4. Notice in both Figs. 3 and 4, there is an equilibrium without credit since $\bar{b} = 0$ is always a solution to (16). This captures the idea that an equilibrium without credit is self-fulfilling and can arise under the expectation that borrowers will not repay their debts in the future.

When $\Lambda \leq \overline{\Lambda}$ a pure credit equilibrium does not exist, and when $i \geq \bar{i}$ money is not valued. Is it possible then, when money is valued $\left(i < \bar{i}\right)$, to have an equilibrium with *both* money and credit? We show formally in the next subsection the answer is yes, so long as record-keeping is imperfect and inflation is in an intermediate range.

4.3. Money-credit equilibrium

In an equilibrium with both money and credit, $\bar{b} > 0$ and $z > 0$. We first derive existence conditions for a money-credit equilibrium when $\Lambda \in (0, 1)$, and then turn to a very special case of

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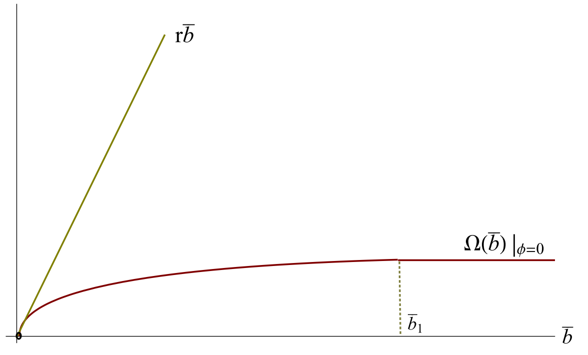

Fig. 4. No credit equilibrium when $\Lambda \le \overline{\Lambda}$.

the model when $\Lambda = 1$. In either case, we verify that two conditions are satisfied for a money-credit equilibrium to exist: *(1)* credit is incentive feasible given money is valued and *(2)* money is valued given a debt limit.

To describe a monetary equilibrium with credit, we further characterize $\Omega(\bar{b})|_{\phi>0}$ by establishing the existence of a critical debt limit above which money is no longer valued. If it exists, it is the value $\bar{b}_0$ that solves (12) at equality with $z = 0$:

$$i = \frac{\sigma(1-\Lambda)\theta}{1-\theta} + \sigma\Lambda\theta S'(\bar{b}_0). \tag{19}$$

For (19) to be well defined, $\bar{b}_0$ must be non-negative and finite. Accordingly, the threshold $\bar{b}_0$ exists if and only if $i \ge \widehat{i} \equiv \frac{\sigma(1-\Lambda)\theta}{1-\theta}$. Notice when $\Lambda = 1$, $\bar{b}_0$ exists for all $i > 0$.

Lemma 3. *Given $\Lambda \in (0, 1)$ and $i > 0$, the following outcomes are possible.*

1. *Suppose $i < \widehat{i}$. Then, $z > 0$ for all $\bar{b} \in [0, \infty)$.*
2. *Suppose $i \ge \widehat{i}$. Then, $z > 0$ for all $\bar{b} \in [0, \bar{b}_0)$ and $z = 0$ otherwise.*

According to Lemma 3, $i < \widehat{i}$ implies that for any debt limit buyers may still value money, and $i \ge \widehat{i}$ implies there exists a critical debt limit above which buyers do not value money anymore. Notice when $i = \widehat{i}$, $\bar{b}_0 = \bar{b}_1$, in which case money can be valued for all $\bar{b} < \bar{b}_1$. Intuitively, when $i \ge \widehat{i}$, even though a subset of sellers only accept cash, buyers may choose not to hold money if credit is sufficiently abundant. Given $i > 0$, notice whenever money is valued, $q < q^*$ and $q^c < q^*$. As is standard in this class of models, buyers are liquidity constrained in any monetary equilibrium except possibly at the Friedman rule. At the Friedman rule, there is a unique equilibrium with positively valued money where credit limits cannot be sustained.

We now turn to verifying the conditions for a money-credit equilibrium to exist. First, given money is valued, there is a positive debt limit if the slope of $\Omega(\bar{b})|_{\phi>0}$ at $\bar{b} = 0$, $\sigma\theta\Lambda S'(z)$, is greater than the slope of $r\bar{b}$, or

$$i\Lambda > r,$$

where we have used the fact that when money is valued, $i = \sigma\theta S'(z)$ at $\bar{b} = 0$. As a result, a necessary condition for credit to be feasible given valued money is

$$i > \frac{r}{\Lambda} \equiv \underline{i}. \tag{20}$$

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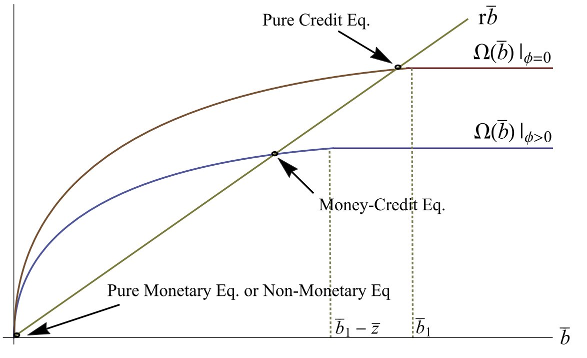

Fig. 5. Money-credit Eq. when $i < \widehat{i}$.

The critical value $\underline{i}$ is the lower bound on the nominal interest rate, above which credit is feasible when money is valued. Intuitively, condition (20) says that the cost of holding money cannot be too low so that buyers would prefer to renege on repayment. At the same time, buyers also have to be patient enough to care about the possibility of future punishment.

Second, given $\bar{b}$, there is an interior solution for real balances if and only if

$$i < \frac{\sigma(1-\Lambda)\theta}{1-\theta} + \sigma\Lambda\theta S'[\bar{b}(i,\Lambda)],$$

where $\bar{b}(i,\Lambda)$ implicitly defines $\bar{b}$ as a monotonic function of i and Λ. Consequently, money is valued given a positive debt limit if

$$i < \widetilde{i}, \tag{21}$$

where $\widetilde{i}$ solves

$$\widetilde{i} = \frac{\sigma(1-\Lambda)\theta}{1-\theta} + \sigma\Lambda\theta S'[\bar{b}(\widetilde{i},\Lambda)]. \tag{22}$$

Notice $\underline{i} < \widetilde{i}$ if and only if $\Lambda > \overline{\Lambda}$, so a necessary condition for a money-credit equilibrium is $\Lambda > \overline{\Lambda}$. Consequently, a necessary and sufficient condition for a money-credit equilibrium to exist is $\Lambda > \overline{\Lambda}$ and $i \in (\underline{i}, \widetilde{i})$.

Figs. 5 and 6 depict two examples of a money-credit equilibrium when $i < \widehat{i}$ and $i > \widehat{i}$, respectively. In both cases, a money-credit equilibrium exists if $\Lambda \in (\overline{\Lambda}, 1)$ and $i \in (\underline{i}, \widetilde{i})$. Recall $\Lambda > \overline{\Lambda}$ is a necessary and sufficient condition for a pure credit equilibrium to be feasible. Moreover, since $\widetilde{i} < \bar{i}$, the necessary and sufficient condition for a pure monetary equilibrium, $i < \bar{i}$, is always satisfied whenever $i < \widetilde{i}$. Consequently, whenever there exists a money-credit equilibrium, there also exists a pure monetary equilibrium and pure credit equilibrium.

4.3.1. *Perfect record-keeping*

We now turn to a special case of the model with perfect record-keeping ($\Lambda = 1$). As before, the cost of default depends on whether or not money is valued. When money is not valued, the right side of (16) is

$$\Omega(\bar{b})|_{\phi=0} = \sigma\theta S(\bar{b}),$$

which is strictly concave if $\bar{b} \in [0, \bar{b}_1)$ and linear otherwise. As before, since a non-monetary equilibrium always exists, $\Omega(\bar{b})|_{\phi=0}$ is defined for all $\bar{b} \in [0, \infty)$.

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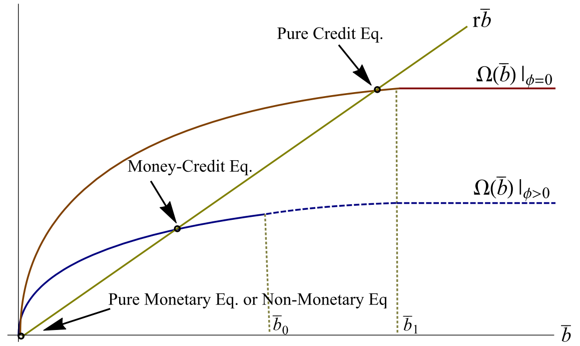

Fig. 6. Money-credit Eq. when $i > \hat{i}$.

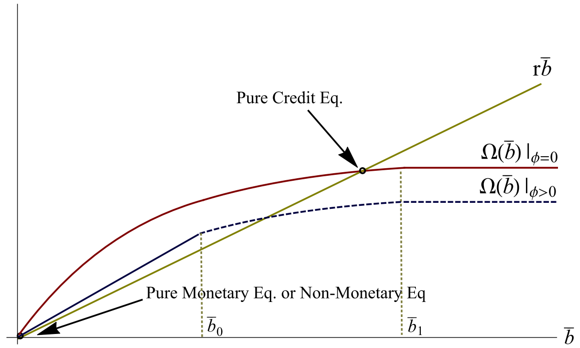

Fig. 7. No money-credit Eq. when $\Lambda = 1$, $r < i < \bar{i}$ (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

When money is valued, (12) and (13) at equality implies a buyer chooses his real balances so that his total wealth is the same as a buyer who defaults: $z + \bar{b} = \tilde{z}$. In that case, the right side of (16) when money is valued and $\Lambda = 1$ is

$$\Omega(\bar{b})|_{\phi>0} = i\bar{b},$$

which is linear with a slope of i for all $\bar{b} \in [0, \bar{b}_0)$. This is represented by the straight blue line in Figs. 7 and 8.

When $\Lambda = 1$, a necessary condition for credit is that the slope of $\Omega(\bar{b})|_{\phi=0}$ at $\bar{b} = 0$ is greater than the slope of $r\bar{b}$. Given money is not valued, credit is feasible if $\sigma\theta S'(0) > r$, or

$$r < \frac{\sigma\theta}{1-\theta} \equiv \bar{i},$$

which is the same necessary condition for a pure credit equilibrium derived in Section 4.2, but with $\Lambda = 1$ in (18). Moreover, there is a positive debt limit where money is not valued if $r < i < \bar{i}$ or $i < r < \bar{i}$, which implies there is at most one positive intersection of $\Omega(\bar{b})|_{\phi=0}$ with $r\bar{b}$. If $i = r < \bar{i}$, the debt limit is indeterminate and there are a continuum of debt limits where $\bar{b} \in [0, \bar{b}_0]$. Finally if $r \geq \bar{i}$, credit is not feasible, and the only solution to (16) is $\bar{b} = 0$.

Can money and credit coexist under perfect record-keeping? We prove in the special case of our model with $\Lambda = 1$, the answer is generally no. While a formal proof is in Appendix A, the reasoning can be seen from (22). Notice when $\Lambda = 1$, $\tilde{i} = \underline{i} = r$, which means the existence

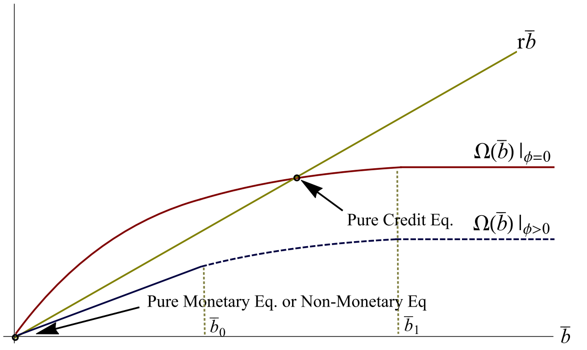

Fig. 8. No money-credit Eq. when $\Lambda = 1$, $i < r < \bar{i}$ (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

region for a money-credit equilibrium, $i \in (\underline{i}, \tilde{i})$, collapses to a single point, $\tilde{i} = \underline{i} = r$. Consequently, a money-credit equilibrium ceases to exist when $\Lambda = 1$ for any $i \neq r$.

In the knife edge case where $\Lambda = 1$ and $i = r < \bar{i}$, the money supply is constant and there are a continuum of debt limits, $\bar{b} \in [0, \bar{b}_0]$, that satisfy (16). In each of these equilibria, allocations are identical in payoffs. In turn, any exogenous change in policy that affects the debt limit is perfectly offset by a change in real balances so that the buyer's total wealth is the same. In that case, money and credit are not *both* needed.

We summarize our main coexistence results for the model with $\Lambda \in (0, 1]$ in the following proposition, where a non-monetary equilibrium without credit always exists.

Proposition 1. *Given $\Lambda \in (0, 1]$ and $i > 0$, the following outcomes are possible.*

1. *When $\Lambda \in (0, \overline{\Lambda}]$, credit is not feasible and a pure monetary equilibrium exists if and only if $i < \bar{i}$.*
2. *When $\Lambda \in (\overline{\Lambda}, 1)$, a money-credit equilibrium exists if and only if $i \in (\underline{i}, \tilde{i})$, and coexists with a pure monetary equilibrium, and a pure credit equilibrium.*
3. *When $\Lambda = 1$, a money-credit equilibrium does not exist for any $i \neq r$.*
   (a) *If $r < i < \bar{i}$ or $i < r < \bar{i}$, there exists a pure monetary equilibrium and a pure credit equilibrium.*
   (b) *If $i < \bar{i}$ and $r \geq \bar{i}$, there exists a pure monetary equilibrium, but not a pure credit equilibrium.*
   (c) *If $i \geq \bar{i}$ and $r < \bar{i}$, there exists a pure credit equilibrium, but not a pure monetary equilibrium.*
   (d) *If $i = r < \bar{i}$, there exists a pure monetary equilibrium, a pure credit equilibrium, and a continuum of equilibria where the debt limit is indeterminate.*

Proposition 1 highlights an important dichotomy between monetary and credit trades when record-keeping is perfect: there can be trades with credit only or trades with money only, but generally not trades with both. This special case also points to the fundamental difficulty of getting money and credit to coexist when all trades are identical and record-keeping is perfect: either only credit is used as money becomes inessential, or only money is used since the incentive to renege on debt repayment is too high. This also captures the insight by Kocherlakota (1998) that there is no social role for money in an economy with perfect record-keeping.

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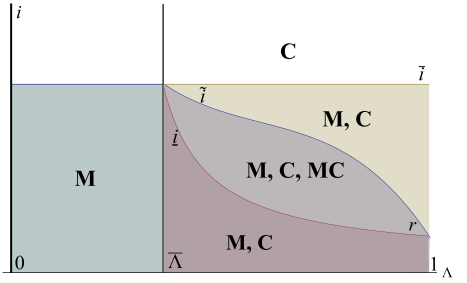

Fig. 9. Typology of equilibria in (Λ, i)-space.

4.3.2. *Comparative statics*

Having characterized existence of a money-credit equilibrium, we now discuss some comparative statics for effects on the debt limit summarized in the table below. In what follows, we focus on equilibria where money is valued and credit is feasible.

$\frac{\partial \bar{b}}{\partial \Lambda}$	$\frac{\partial \bar{b}}{\partial \sigma}$	$\frac{\partial \bar{b}}{\partial i}$	$\frac{\partial \bar{b}}{\partial r}$	$\frac{\partial \bar{b}}{\partial \theta}$
$+$	$+$	$+$	$-$	$+$

An increase in the fraction of the sellers with access to record-keeping, Λ, increases the right side of (16), which shifts up both $\Omega(\bar{b})|_{\phi=0}$ and $\Omega(\bar{b})|_{\phi>0}$ and induces an increase in $\bar{b}$. When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the debt limit. The increase in Λ can be high enough so that credit drives money out of circulation. When $\Lambda = 1$ and $i \neq r$, a money-credit equilibrium disappears, in which case there can only be a pure monetary equilibrium or a pure credit equilibrium.

An increase in inflation generates a similar qualitative effect. It has two effects in this model: the first is the usual effect on reducing the purchasing power of money, which reduces trade and hence welfare; the second effect is on the incentives to default. Intuitively, an increase in the inflation tax relaxes the credit constraint by increasing the cost of default, since defaulters need to bring enough money to finance their consumption.

To summarize, the debt limit depends on the fraction of credit trades, the extent of trading frictions, the rate of return on money, agents' patience, and the buyer's bargaining power. The larger the fraction of sellers that accept credit, the lower the rate of return on money, or the more patient agents become, the less likely the credit constraint will be binding. In these cases, the buyer can credibly promise to repay more, which induces cooperation in credit arrangements thereby relaxing the debt limit.

4.4. *Multiple equilibria*

A key feature of the model is the presence of multiple steady-state equilibria where different transactions patterns coexist. In Fig. 9, we illustrate the typology of active steady-state equilibria in (Λ, i)-space. The active steady-states correspond to a pure credit equilibrium (C), a pure monetary equilibrium (M), and a mixed equilibrium where both money and credit are used (MC). As shown in Section 4.3, the existence of the MC region requires both heterogeneous record-keeping across sellers and imperfect enforcement. With perfect record-keeping ($\Lambda = 1$), a money-credit equilibrium generally does not exist, as shown in Proposition 1. And with perfect enforcement, our model reduces to the model in Nosal and Rocheteau (2011), where there is no equilibrium where a buyer uses both money and credit in a DM transaction.

5. Endogenous record-keeping

We now consider the choice of accepting credit by making Λ endogenous. In order to accept credit, sellers must invest ex-ante in a costly record-keeping technology that records and authenticates an IOU proposed by the buyer.[9] The per-period cost of this investment in terms of utility is $\kappa \geq 0$, drawn from a cumulative distribution $F(\kappa) : \mathbb{R}_+ \to [0, 1]$. Sellers are heterogeneous according to their record-keeping cost and are indexed by κ. For instance, κ captures an explicit adoption cost component and an implicit opportunity or cognitive cost component that varies across sellers in different industries or sectors.[10]

At the beginning of each period, sellers decide whether or not to invest in the record-keeping technology. When making this decision, sellers take as given buyers' choice of real balances and the debt limit. The seller's problem is given by

$$\max\{-\kappa + \sigma(1-\theta)S(z+\bar{b}), \sigma(1-\theta)S(z)\}. \tag{23}$$

According to (23), a seller who invests incurs the disutility cost $\kappa > 0$ that allows him to extend a loan to the buyer. In that case, the seller extracts a constant fraction $(1-\theta)$ of the total surplus, $S(z+\bar{b})$. If the seller does not invest, then he can only accept money, and gets $(1-\theta)$ of $S(z)$. Since total surplus is increasing in the buyer's wealth, the total surplus from accepting credit is at least as large as the total surplus from not accepting credit, $S(z+\bar{b}) > S(z)$.

In addition, there exists a threshold for the record-keeping cost, $\bar{\kappa}$, below which sellers invest and above which they do not invest. From (23), this threshold is given by

$$\bar{\kappa} \equiv \sigma(1-\theta)[S(z+\bar{b}) - S(z)], \tag{24}$$

and gives an individual seller's expected benefit of accepting credit. Given κ, let $\lambda(\kappa) \in [0, 1]$ denote an individual seller's decision to invest. This decision problem is given by

$$\lambda(\kappa) = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \text{if } \kappa \begin{cases} < \\ = \\ > \end{cases} \bar{\kappa}. \tag{25}$$

According to (25), all sellers with $\kappa < \bar{\kappa}$ invest since the benefit exceeds the cost; sellers with $\kappa > \bar{\kappa}$ do not invest; and any seller with $\kappa = \bar{\kappa}$ invests with an arbitrary probability since they are

[9] This cost can also reflect issues of fraud and information problems that permeate the credit industry such as credit card fraud, identity theft, and the need to secure confidential information. See Li et al. (2012) for a recent model of fraud.

[10] In addition to being a plausible assumption in reality (see e.g. Garcia-Swartz et al. (2006)), another reason we assume κ potentially differs across sellers is because when all sellers have the same κ, an interior equilibrium where a positive measure of sellers invest is not stable in the following sense. Suppose $\Lambda \in (0, 1)$ is an equilibrium when all sellers have the same κ. If we perturb the equilibrium slightly to $\Lambda + \epsilon$ for an arbitrarily small $\epsilon > 0$ ($\epsilon < 0$), then it is a best response for all sellers (no sellers) to invest. In addition, while we assume here that κ is a fixed flow cost, having a one-time cost may be more suitable in some applications but does not affect the key insights of our analysis. See also Lester et al. (2012) and Zhang (2014) for a related discussion and further applications.

indifferent. Consequently, the aggregate measure of sellers that invest is given by the measure of sellers with $\kappa \leq \bar{\kappa}$:

$$\Lambda \equiv \int_0^\infty \lambda(\kappa) dF(\kappa) = F(\bar{\kappa}). \tag{26}$$

Definition 2. A stationary equilibrium with endogenous record-keeping is a list $(q, q^c, z, \widetilde{z}, \bar{b}, \lambda(\kappa), \Lambda)$ that solves (8), (9), (12), (13), (16), (25), and (26).

Equilibrium with endogenous record-keeping can be determined recursively. We first determine the debt limit for a given measure of sellers that invest and value of money, as in Section 4. Next, we determine sellers' investment decisions given the debt limit and value of money. We assume all sellers have the same expectations about the value of money and these expectations are rational and consistent with buyers' portfolio choices. Having jointly determined $(\bar{b}, \Lambda)$, the determination of all other endogenous variables follow directly from the analysis in the previous sections.

5.1. Equilibrium debt limit

In the following lemma, we summarize how the debt limit depends on the measure of sellers with access to record-keeping, Λ, and value of money, ϕ. Our characterization below focuses on positive solutions to the debt limit (we know that $\bar{b} = 0$ is always a solution to (16) for any Λ, whether or not money is valued). To simplify the presentation, we assume $i \geq \widehat{i}$ so the threshold $\bar{b}_0$ exists; the analysis when $i < \widehat{i}$ is similar except we would replace $\bar{b}_0$ with $\bar{b}_1 - \bar{z}$.

Lemma 4. *Define Λ_0 as the value of Λ when $\bar{b} = \bar{b}_0$ solves (16), and Λ_1 as the value of Λ when $\bar{b} = \bar{b}_1$ solves (16).*

1. *When money is not valued, the debt limit, $\bar{b}(\Lambda)|_{\phi=0}$, is strictly increasing and convex in Λ for all $\Lambda \in (\overline{\Lambda}, \Lambda_1)$ and is increasing and linear for all $\Lambda \in [\Lambda_1, 1)$.*
2. *When money is valued, the debt limit, $\bar{b}(\Lambda)|_{\phi>0}$, is strictly increasing and convex in Λ for all $\Lambda \in (\frac{r}{i}, \Lambda_0)$.*

In Lemma 4, $\bar{b}(\Lambda)|_{\phi=0}$ is defined if and only if credit is feasible, or $\Lambda > \overline{\Lambda}$. Similarly, $\bar{b}(\Lambda)|_{\phi>0}$ is defined for $\Lambda \in (\frac{r}{i}, \Lambda_0)$ since the existence condition for a money-credit equilibrium, $i \in (\underline{i}, \widetilde{i})$, is satisfied if and only if $\Lambda \in (\frac{r}{i}, \Lambda_0)$.

5.2. Equilibrium measure of sellers that invest in record-keeping

Given a debt limit and choice of real balances by buyers, sellers decide whether to invest in the record-keeping technology. For an individual seller, this decision is determined by (25) and hence depends on the expected benefit of investing, $\bar{\kappa}$.

Depending on the distribution of the cost function, $F(\kappa)$, there can be any number of solutions to Λ, though in our analysis below we give simple conditions to ensure an interior equilibrium where a positive measure of sellers invest. Indeed a special case is when the record-keeping cost is zero for a constant fraction Λ of sellers and infinite for the rest, as in Section 4, which implies

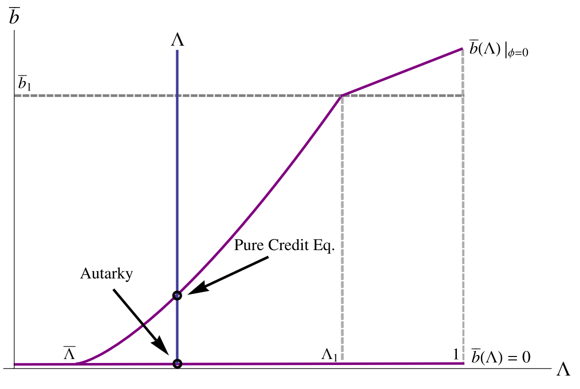

Fig. 10. $\bar{b}$ when money not valued ($\phi = 0$).

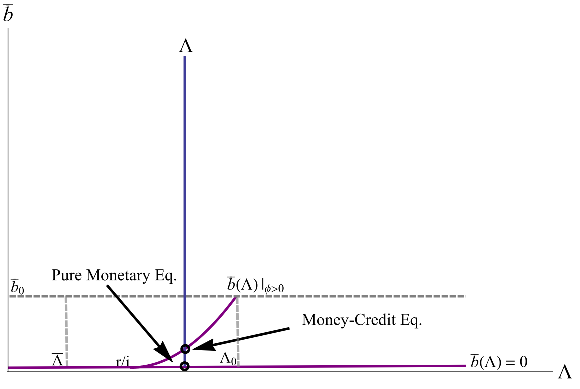

Fig. 11. $\bar{b}$ when money is valued ($\phi > 0$).

the measure of sellers investing is constant at Λ for any debt limit. While it is clearly desirable to proceed with more general cost functions where $F(\kappa)$ is non-degenerate so the measure of sellers investing can depend on the debt limit, we present this simple example first to illustrate our basic methodology. In that case, the fraction of sellers accepting credit, Λ, is represented by the vertical line in Figs. 10 and 11 when money is not valued ($\phi = 0$), and when money is valued ($\phi > 0$), respectively. Since there is always at least one intersection of Λ with either $\bar{b}(\Lambda)|_{\phi=0}$, $\bar{b}(\Lambda)|_{\phi>0}$, or $\bar{b}(\Lambda) = 0$ in Figs. 10 and 11, an equilibrium with endogenous debt limits and record-keeping exists, and for any $\Lambda > \bar{\Lambda}$, we know from Section 4 that there exists at least one positive solution to the debt limit.

We now turn to the more general case where $F(\kappa)$ is possibly non-degenerate. To ensure an interior solution, we assume κ is zero for a positive measure of sellers and that κ is arbitrarily high for a positive measure of sellers. As with our characterization of the debt limit, here we distinguish two cases for the seller's benefit of investing: (*1*) when money is not valued ($\phi = 0$) and (*2*) when money is valued ($\phi > 0$). These are given respectively by

$$\bar{\kappa}(\bar{b})|_{\phi=0} \equiv \sigma(1-\theta)S(\bar{b}),$$

$$\bar{\kappa}(\bar{b})|_{\phi>0} \equiv \sigma(1-\theta)[S(z+\bar{b}) - S(z)],$$

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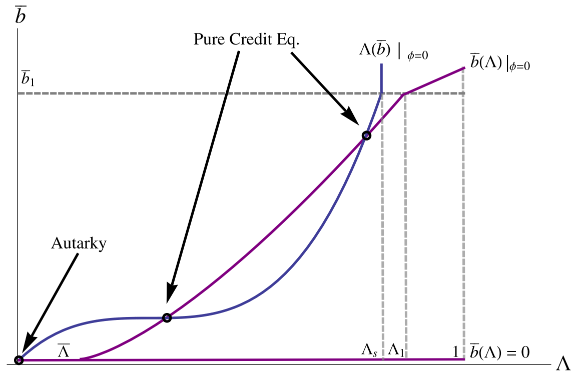

Fig. 12. $(\bar{b}, \Lambda)$ when money not valued $(\phi = 0)$.

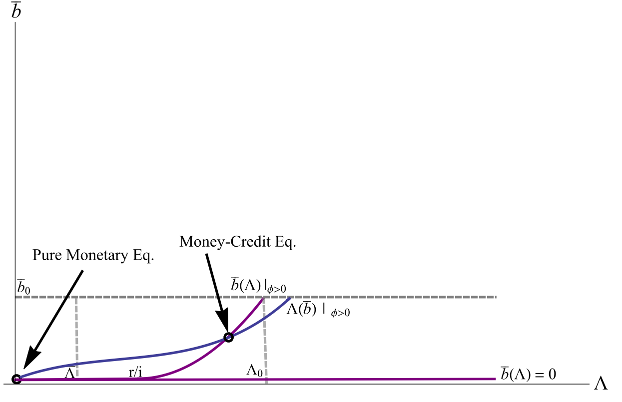

Fig. 13. $(\bar{b}, \Lambda)$ when money is valued $(\phi > 0)$.

which are both increasing in the loan size, $\bar{b}$. Given $\bar{\kappa}(\bar{b})|_{\phi=0}$ and $\bar{\kappa}(\bar{b})|_{\phi>0}$, we denote the corresponding measure of sellers that invest by $\Lambda(\bar{b})|_{\phi=0}$ and $\Lambda(\bar{b})|_{\phi>0}$, respectively. The following lemma summarizes how sellers' aggregate investment decisions depends on the debt limit and value of money.

Lemma 5. *Define Λ_s as the measure of sellers that invest when $\bar{b} = \bar{b}_1$.*

1. *When money is not valued, the measure of sellers that invest, $\Lambda(\bar{b})|_{\phi=0}$, is strictly increasing in $\bar{b}$ for all $\bar{b} \in [0, \bar{b}_1)$ and constant at $\Lambda_s \le 1$ for all $\bar{b} \ge \bar{b}_1$.*
2. *When money is valued, the measure of sellers that invest, $\Lambda(\bar{b})|_{\phi>0}$, is strictly increasing in $\bar{b}$ for all $\bar{b} \in [0, \bar{b}_0)$.*

Together, Lemma 4 and Lemma 5 help determine an equilibrium with endogenous record-keeping summarized in the following proposition.

Proposition 2. *Suppose $F(\kappa) : \mathbb{R}_+ \to [0, 1]$ is such that κ is zero for a positive measure of sellers and κ is arbitrarily high for a positive measure of sellers. Equilibrium with endogenous* *record-keeping exists, and given $i > 0$, multiple pure credit or money-credit steady-states are possible.*

An example of Proposition 2 where $F(\kappa)$ is non-degenerate is illustrated in Figs. 12 and 13 when money is not valued ($\phi = 0$) and when money is valued ($\phi > 0$), respectively.[11] Hence, when $i \in (r, \bar{i})$ so that both $\phi = 0$ and $\phi > 0$ are possible, the correspondences for $\bar{b}$ and Λ can intersect multiple times, corresponding to multiple steady-state equilibria with endogenous debt limits and record-keeping. Indeed, if we superimpose Figs. 12 and 13, which corresponds to outcomes where $i \in (r, \bar{i})$, there is *(i)* a pure monetary equilibrium where no sellers accept credit, *(ii)* a money-credit equilibrium where a fraction of sellers accept both money and credit, *(iii)* two pure credit equilibria where money is not valued and a fraction of sellers accept credit, and *(iv)* a non-monetary equilibrium without credit, or autarky.

More generally, while there can be any number solutions to Λ for an arbitrary $F(\kappa)$, we know that for any Λ, we can determine the debt limit according to Lemma 4. Then once $(\bar{b}, \Lambda)$ are jointly determined, we can easily obtain all other endogenous variables, namely $(q, q^c, z, \tilde{z})$ from Definition 2. Given $i > 0$, so that we are not at the Friedman rule, it is also possible to have multiple steady-state equilibria with different values for $(\Lambda, \bar{b})$ and in turn, different configurations in the usage of money and credit.

The multiplicity of equilibria arises through the general equilibrium effects in the trading environment that produce strategic complementarities between buyers' and sellers' decisions.[12] As is evident from the upward sloping curves in Fig. 13, the measure of sellers who invest is increasing in the debt limit and vice versa. When more sellers invest in the costly record-keeping technology, the gain for buyers from using credit also increase. As default becomes more costly, the incentive to renege falls which raises the debt limit. At the same time, when more sellers accept credit, then money is needed in a smaller fraction of matches. So long as holding money is not too costly, buyers carry fewer real balances. This gives sellers an even greater incentive to accept credit, which raises the debt limit and further reduces the buyer's real balances.[13]

6. Welfare

So far, we have seen that for the same fundamentals and monetary policy, the economy may end up in a pure credit, pure monetary, or money-credit equilibrium. How do these equilibria compare in terms of social welfare? In addition, how does a one-time changes in monetary policy affect welfare for each type of equilibrium?

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[11] The examples in Figs. 12 and 13 assume $u(q) = 2\sqrt{q}$, $c(q) = q$, $F(\kappa) = \left(\frac{1}{1+\exp^{-(\kappa-\mu)}}\right)$, $\sigma = 0.5$, $\theta = 0.8$, $r = 0.02$, $\mu = 2$, and $i = 0.3$. The equilibrium values in Figs. 12 and 13 are, respectively, $(\Lambda, \bar{b}) = \{(0,0), (0.19, 0.11), (0.76, 0.81)\}$ and $(\Lambda, \bar{b}) = \{(0,0), (0.22, 0.12)\}$.

[12] Strategic complementarities between the seller's decision to invest and the buyer's choice of real balances is also present under perfect enforcement as in Nosal and Rocheteau (2011). The complementarities in our model captures both an extensive margin effect (on the number of credit trades) and intensive margin effect (on the amount borrowed per trade), whereas the latter intensive margin channel is absent in Nosal and Rocheteau (2011).

[13] The complementarities in this paper are also similar to the complementarities between sellers' investment decisions and buyers' portfolios in Lester et al. (2012). In our model, there are also complementarities between the actions of current and future agents which is present even in the model with exogenous record-keeping: if more future sellers accept credit, default by buyers becomes more costly, relaxing the debt limit today.

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Table 1
Welfare across steady-state equilibria.

Equilibrium	Steady-state welfare
Pure Monetary Eq.	$\mathcal{W}^m = \sigma S(z)$
Pure Credit Eq.	$\mathcal{W}^c = \sigma \Lambda S(\bar{b}) - k$
Money-Credit Eq.	$\mathcal{W}^{mc} = \sigma[\Lambda S(z+\bar{b}) + (1-\Lambda)S(z)] - k$

To answer these questions, we now turn to examining the model's normative properties by comparing the types of steady-state equilibria in terms of social welfare. Social welfare is measured as the steady-state sum of buyers' and sellers' utilities:

$$\mathcal{W} \equiv \sigma[\Lambda S(z+\bar{b}) + (1-\Lambda)S(z)] - k \tag{27}$$

where $k \equiv \int_0^{\bar{\kappa}} \kappa dF(\kappa)$ is the aggregate record-keeping cost averaged across individual sellers. Table 1 summarizes social welfare in the three types of active equilibria discussed in the previous sections.

Fig. 14 plots an example of welfare in the three types of equilibria as a function of the cost of holding money, i.[14] At $i = 0$, which is the Friedman rule, Kalai bargaining implies an efficient economy can run without credit. In that case, welfare in a pure monetary equilibrium is at its maximum, $\mathcal{W}^{\max} \equiv \sigma[u(q^*) - c(q^*)]$, and strictly dominates welfare in a pure credit equilibrium since sellers must incur the real cost of technological adoption. Hence, when record-keeping is costly, a pure credit equilibrium is socially inefficient even when $r \le r^*$ so that credit alone can achieve the first-best. As i increases, it may be possible that $\mathcal{W}^m < \mathcal{W}^c$ which occurs if $i > i_c$, where i_c is the value of i such that $\mathcal{W}^m = \mathcal{W}^c$:

$$S[z(i_c)] = \Lambda S(\bar{b}) - k.$$

We can also have $\mathcal{W}^m < \mathcal{W}^c$ if the aggregate record-keeping cost is low enough—that is, if $k < \sigma[\Lambda S(\bar{b}) - S[z(i)]]$, or if inflation is high enough, as illustrated in Fig. 14. However, the welfare-dominated monetary equilibrium may still prevail due to a rent-sharing externality: since sellers incur the full cost of technological adoption but only obtain a fraction $(1-\theta)$ of the total surplus, they fail to internalize the full benefit of accepting credit. Consequently, there can be coordination failures and excess inertia in the decision to accept credit, in which case the economy can still end up in the Pareto-inferior monetary equilibrium.

We also compare welfare in a money-credit economy with welfare in a pure monetary economy and pure credit economy. Recall the necessary condition for a money-credit equilibrium is $i \in (\underline{i}, \tilde{i})$. When $i = \underline{i}$, credit is no longer feasible if money is valued, in which case $\mathcal{W}^{mc} = \sigma S(z) = \mathcal{W}^m$. Alternatively when $i = \tilde{i}$, money is no longer valued if there is credit, in which case $\mathcal{W}^{mc} = \sigma \Lambda S(\bar{b}) - k = \mathcal{W}^c$. In the example in Fig. 14, we therefore have $\mathcal{W}^{mc} > \mathcal{W}^m$ and $\mathcal{W}^{mc} > \mathcal{W}^c$ for $i \in (\underline{i}, \tilde{i})$ given the threshold i_c exists. To be clear however, the welfare rankings in Fig. 14 are not generic and will depend on fundamentals like the record-keeping cost. For instance, if κ is very high, then for $i \in (\underline{i}, \tilde{i})$, the surplus from accepting credit can be low enough that $\mathcal{W}^m > \mathcal{W}^{mc} > \mathcal{W}^c$.

[14] The example in Fig. 14 assumes $u(q) = 2\sqrt{q}$, $c(q) = q$, $F(\kappa) = \left(\frac{1}{1+\exp^{-(\kappa-\mu)}}\right)$, $\sigma = 0.5$, $\theta = 0.8$, $r = 0.02$, $\mu = 2$, and varies i.

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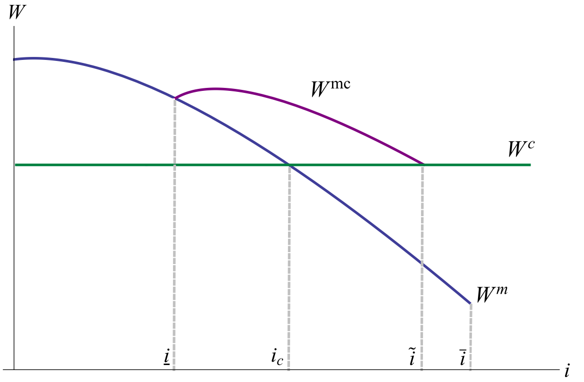

Fig. 14. Welfare in pure monetary, money-credit, and pure credit Eq. (example).

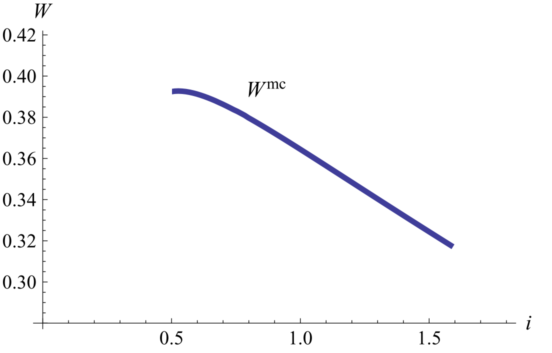

Fig. 15. $\mathcal{W}^{mc}$ when $\Lambda > \widehat{\Lambda}$ (example).

6.1. *Effects of inflation on welfare*

To study the welfare effects of a change in inflation for the three types of payment regimes, we now do some comparative statics of i. The presence of multiple equilibria for the same fundamentals makes the choice of optimal policy difficult to analyze in full generality since we must deal with the issue of equilibrium selection. In regions with multiplicity, we assume agents coordinate on a particular equilibrium and then analyze the effects of (perfectly anticipated) inflation in that equilibrium. We also begin by taking $\Lambda \in (0, 1)$ as given.

In a pure credit equilibrium, inflation has no effect on welfare for any $i > 0$. This is illustrated by the horizontal green line in Fig. 14. In a pure monetary equilibrium, which is possible if $i < \bar{i}$, welfare is decreasing in the inflation rate: $\frac{d\mathcal{W}_m}{di} = \sigma S'(z)\frac{dz}{di} < 0$ since $\frac{dz}{di} < 0$ for any $i < \bar{i}$. If lump-sum taxes can be enforced, then optimal policy corresponds to the Friedman rule. Indeed when $i = 0$, holding money is costless so there is no need for credit. In that case, deflation completely crowds out credit as money alone is enough to finance the first best.[15]

[15] As we show in Appendix A, the Friedman rule may not be feasible since it requires taxation and individuals may choose to renege on their tax obligation if it is too high. In that case, the first best can only be achieved with credit.

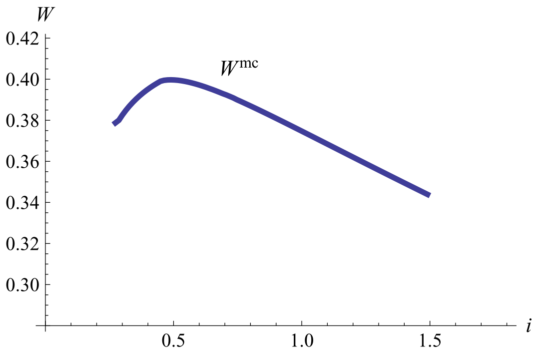

Fig. 16. $\mathcal{W}^{mc}$ when $\Lambda < \widehat{\Lambda}$ (example).

In a money-credit equilibrium, the effect of inflation on welfare is given by

$$\frac{d\mathcal{W}_{mc}}{di} = \sigma \Lambda S'(z+\bar{b}) \left[\frac{dz}{di} + \left(\frac{dz}{d\bar{b}} + 1 \right) \frac{d\bar{b}}{di} \right] + \sigma(1-\Lambda) S'(z) \frac{dz}{di}, \tag{28}$$

where $\frac{dz}{di} < 0$, $\frac{dz}{d\bar{b}} \in (-1, 0)$, and $\frac{d\bar{b}}{di} > 0$. In general, the sign of (28) is ambiguous and depends on the relative magnitudes of two counteracting effects: the effect of inflation on real balances ($\frac{dz}{di} < 0$) and the effect of inflation on the debt limit ($\frac{d\bar{b}}{di} > 0$). In Appendix A, we derive a sufficient condition for inflation to have a negative effect on welfare. Specifically, when $\Lambda > \widehat{\Lambda} \equiv \frac{S''(z)}{S''(z)+S''(z+\bar{b})}$, $\frac{d\mathcal{W}^{mc}}{di} < 0$. Fig. 15 plots an example illustrating inflation's negative effect on $\mathcal{W}^{mc}$. Intuitively, when Λ is high, the marginal surplus of using credit is low since the debt limit is high whereas the marginal surplus of using money is very high in the $(1-\Lambda)$ cash-only transactions. Hence, when $\Lambda > \widehat{\Lambda}$, an increase in inflation does not increase the surplus as much in money-credit trades as it reduces the surplus in money-only trades. However, when $\Lambda \in (0, \widehat{\Lambda})$, inflation can have a positive effect on welfare, as in Fig. 16. While inflation unambiguously decreases welfare in the $(1-\Lambda)$ trades involving money only, inflation relaxes borrowing constraints in the Λ trades with both money and credit. So long as Λ is not too large, the latter effect may dominate so that an increase in inflation can raise aggregate welfare.

When Λ is endogenous, the positive effect of inflation on welfare is amplified and generates additional feedback effects. Since an increase in inflation raises the debt limit, sellers have a greater incentive to accept credit, which further relaxes the debt limit and the amount borrowed by the buyer. When the debt limit relaxes to the point where buyers are no longer liquidity constrained, money is no longer valued. In that case, agents can still trade the first-best level of output even when monetary authorities do not implement the Friedman rule. However, such an equilibrium is not socially efficient since sellers must still incur the real cost of technological adoption.

To summarize our welfare results, Figs. 14 and 16 show examples where an increase in inflation raises welfare in a money-credit economy. Globally however, maximum welfare in a money-credit economy is strictly dominated by welfare in a pure monetary economy at the Friedman rule, as illustrated in Fig. 14. In that case, money works too well and there is no social role for credit. The Friedman rule remains the globally optimal monetary policy, which implements the first-best allocation and saves society on record-keeping costs.

7. Conclusions

As many economies increasingly rely on credit both as a payment instrument and a means for borrowing, it is increasingly important to understand how individuals substitute between cash and credit. Despite the increasing availability of unsecured lending such as credit cards loans, consumers still demand paper currency and liquid assets for a sizeable share of transactions. That money and credit coexist appears to be the norm across many economies, and one goal of this paper is to delve deeper into understanding why.

To that end, we build a simple model of money and credit that integrates two deep frictions motivating payment decisions: imperfect record-keeping and limited commitment. In order to capture the two-sided nature of actual payment systems, we jointly model the acceptability of credit by merchants and the debt repayment decision by consumers. So long as record-keeping is imperfect, money and credit coexist for a range of parameters. However, in a special case of our model with perfect record-keeping, the classic Kocherlakota (1998) wisdom appears: when credit is feasible, there is no social role for money, and when money is valued, credit cannot be sustained.

Our theory also highlights a strategic complementarity between buyers' credit limits and sellers' decision to accept credit. Multiple steady-state equilibria and coordination failures can therefore arise due to the two-sided nature of the retail payment system. This potential for coordination failures also raises new concerns for policymakers as economies with similar technologies, institutions, and policies can still end up with very different payment systems.

Appendix A

A.1. Concavity of buyer's objective function

The buyer's objective function is

$$\Psi(z) = -iz + \sigma\theta\Lambda[u(q^c) - c(q^c)] + \sigma\theta(1-\Lambda)[u(q) - c(q)],$$

where q^c and q are given by $z + \bar{b} = (1-\theta)u(q^c) + \theta c(q^c)$ and $z = (1-\theta)u(q) + \theta c(q)$, respectively, and $\bar{b}$ is given by (16). The partial derivative of the buyer's objective function with respect to the choice of real balances is

$$\Psi'(z) = -i + \sigma\theta\Lambda\left[\frac{u'(q^c) - c'(q^c)}{\theta c'(q^c) + (1-\theta)u'(q^c)}\right] + \sigma\theta(1-\Lambda)\left[\frac{u'(q) - c'(q)}{\theta c'(q) + (1-\theta)u'(q)}\right],$$

if $z + \bar{b} < (1-\theta)u(q^*) + \theta c(q^*)$. Consider the case where $z + \bar{b} < (1-\theta)u(q^*) + \theta c(q^*)$ and hence $q^c < q^*$ and $q < q^*$. The second partial derivative is

$$\Psi'' = \sigma\theta[\Lambda\Delta^c + (1-\Lambda)\Delta] < 0,$$

where $\Delta^c = \frac{u''(q^c)c'(q^c) - u'(q^c)c''(q^c)}{[\theta c'(q^c) + (1-\theta)u'(q^c)]^3} < 0$ and $\Delta = \frac{u''(q)c'(q) - u'(q)c''(q)}{[\theta c'(q) + (1-\theta)u'(q)]^3} < 0$. Hence for all $(z, \bar{b})$ such that $\bar{b}$ solves (16) and $z + \bar{b} < (1-\theta)u(q^*) + \theta c(q^*)$, the objective function $\Psi(z)$ is strictly concave. □

Proof of Lemma 2.
(1) When money is not valued, the flow cost of default is given by

$$\Omega(\bar{b})|_{\phi=0} = \sigma\Lambda\theta S(\bar{b}).$$

(a). If $\bar{b}=0$, $\Omega(0)|_{\phi=0}=\sigma\Lambda\theta S(0)=0$.
(b). The slope of $\Omega(\bar{b})|_{\phi=0}$ at $\bar{b}=0$ is

$$\Omega'(0)|_{\phi=0}=\sigma\theta\Lambda S'(0)=\frac{\sigma\Lambda\theta}{1-\theta}>0.$$

(c). The slope of $\Omega(\bar{b})|_{\phi=0}$ is given by

$$\Omega'(\bar{b})|_{\phi=0}=\sigma\theta\Lambda S'(\bar{b})>0$$

for all $\bar{b}<\bar{b}_1$. Otherwise, $\Omega'(\bar{b})|_{\phi=0}=0$.
(d). $\Omega(\bar{b})|_{\phi=0}$ is strictly increasing and concave for all $\bar{b}<\bar{b}_1$ since

$$\Omega'(\bar{b})|_{\phi=0}=\sigma\theta\Lambda S'(\bar{b})>0$$

and

$$\Omega''(\bar{b})|_{\phi=0}=\sigma\theta\Lambda S''(\bar{b})<0$$

for all $\bar{b}<\bar{b}_1$. Otherwise, $\Omega(\bar{b})|_{\phi=0}$ is constant at $\sigma\theta\Lambda S^*$ since $\Omega'(\bar{b})|_{\phi=0}=0$ for all $\bar{b}\geq\bar{b}_1$.
(2) When money is valued, the flow cost of default is given by

$$\Omega(\bar{b})|_{\phi>0}\equiv\max_{z>0}\left\{-iz+\sigma\theta\left[(1-\Lambda)S(z)+\Lambda S\left(z+\bar{b}\right)\right]\right\}-\max_{\widetilde{z}>0}\{-i\widetilde{z}+\sigma\theta S(\widetilde{z})\}.$$

(a). If $\bar{b}=0$, then $z=\widetilde{z}$ since

$$-iz+\sigma\theta S(z)=-i\widetilde{z}+\sigma\theta S(\widetilde{z}).$$

Consequently, $\Omega(0)|_{\phi>0}=0$.
(b). The slope of $\Omega(\bar{b})|_{\phi>0}$ at $\bar{b}=0$ is

$$\Omega'(0)|_{\phi>0}=\sigma\theta\Lambda S'(z)=i\Lambda>0,$$

where we have used the fact that at $\bar{b}=0$, a necessary condition for valued money is $i=\sigma\theta S'(z)$. Moreover, notice that $\Omega'(0)|_{\phi>0}<\Omega'(0)|_{\phi=0}$ if and only if $i<\frac{\sigma\theta}{1-\theta}$.
(c). The slope of $\Omega(\bar{b})|_{\phi>0}$ is given by

$$\Omega'(\bar{b})|_{\phi>0}=\sigma\theta\Lambda S'(z+\bar{b})>0,$$

for all $\bar{b}<\bar{b}_1-\bar{z}$.
(d). $\Omega(\bar{b})|_{\phi>0}$ is strictly increasing and concave for all $\bar{b}<\bar{b}_1-\bar{z}$ since

$$\Omega'(\bar{b})|_{\phi>0}=\sigma\theta\Lambda S'(z+\bar{b})>0$$

and

$$\Omega''(\bar{b})|_{\phi>0}=\sigma\theta\Lambda S''(z+\bar{b})<0$$

for all $\bar{b}<\bar{b}_1-\bar{z}$. □

Proof of Lemma 3.
(1) Suppose $i<\widehat{i}$. Then $\bar{b}_0$ does not exist and money is valued for any $\bar{b}\geq 0$. To verify, we check the conditions for money to be valued given $\bar{b}=0$ and $\bar{b}>0$. First, money is valued given $\bar{b}=0$ if and only if

$$i < \frac{\sigma\theta}{1-\theta} \equiv \bar{i}.$$

Since $\widehat{i} < \bar{i}$ whenever $\Lambda \in (0, 1)$, $i < \widehat{i}$ implies the condition $i < \bar{i}$ is satisfied. Second, money is valued given $\bar{b} > 0$ if and only if $i < \widetilde{i}$ where $\widetilde{i}$ solves

$$\widetilde{i} = \frac{\sigma(1-\Lambda)\theta}{1-\theta} + \sigma\Lambda\theta S'[\bar{b}(\widetilde{i}, \Lambda)]. \tag{A.1}$$

Similarly, since $\widehat{i} < \widetilde{i}$ whenever $\Lambda \in (0, 1)$, $i < \widehat{i}$ implies the condition $i < \widetilde{i}$ is satisfied.
(2) Suppose $i \geq \widehat{i}$. By construction, money is valued if and only if $\bar{b} < \bar{b}_0$. □

Proof of Proposition 1.
(1) The condition for money to be valued when there is no credit is $i < \bar{i}$, and the condition for credit to be feasible is $\Lambda > \overline{\Lambda}$.
(2) The derivation of the necessary condition for a money-credit equilibrium, $i \in (\underline{i}, \widetilde{i})$ appears in the main text. Given $\Lambda \in (0, 1)$, that a pure monetary equilibrium exists whenever there is an equilibrium with both money and credit can be verified by noting the condition for a monetary equilibrium, $i < \bar{i}$, is always satisfied if $i \in (\underline{i}, \widetilde{i})$ and that there is always a solution to (16) with $\bar{b} = 0$. To show that a pure credit equilibrium will also exist when $i \in (\underline{i}, \widetilde{i})$, recall that the necessary condition for credit given $z = \widetilde{z} = 0$ is $r < \sigma\Lambda\frac{\theta}{1-\theta}$, which is equivalent to the condition $\Lambda > \overline{\Lambda}$ or $\underline{i} < \bar{i}$.
(3) When $\Lambda = 1$, here we show that a money-credit equilibrium does not exist for any $i \neq r$. Given $\Lambda = 1$ and money is valued, (12) and (13) imply $i = \sigma\theta S'[q(z + \bar{b})] = \sigma\theta S'[q(\widetilde{z})]$, or $q(z + \bar{b}) = q(\widetilde{z})$. Then since $z + \bar{b} = \widetilde{z} = (1-\theta)u(q) + \theta c(q)$ from the bargaining solution, the right side of the debt limit (16) becomes $-i[z - \widetilde{z}] = -i[\widetilde{z} - \bar{b} - \widetilde{z}] = i\bar{b}$. Consequently, (16) implies that $r\bar{b} = i\bar{b}$, or $\bar{b} = 0$ if $i \neq r$. Hence, when $\Lambda = 1$ and $i \neq r$, buyers obtain the same surplus whether or not they default, in which case there cannot exist a positive debt limit that supports voluntary debt repayment. If $\Lambda = 1$ and $i = r$, any $\bar{b} \in [0, \bar{b}_0]$ is a solution.
(a). If $r < i < \bar{i}$ or $i \leq r < \bar{i}$, it is straightforward to verify a pure monetary equilibrium coexists with a pure credit equilibrium. First, a pure monetary exists if $i < \bar{i}$ for all Λ, which is satisfied. Second, a pure credit equilibrium exists if $r < \bar{i}$ for $\Lambda = 1$, which is also satisfied.
(b). If $i < \bar{i}$ and $r \geq \bar{i}$, money is valued given there is no credit and the necessary condition for a pure credit equilibrium is violated, and hence credit is not feasible.
(c). If $i \geq \bar{i}$ and $r < \bar{i}$, credit is feasible since the necessary condition for a pure credit equilibrium is satisfied, but money is not valued.
(d). If $i = r < \bar{i}$, the conditions for a pure monetary equilibrium and a pure credit equilibrium are satisfied. In addition, given money is valued, the debt limit is indeterminate since there are a continuum of debt limits, $\bar{b} \in [0, \bar{b}_0]$, that satisfy (16). □

Proof of Lemma 4. First, to show the debt limit is increasing in Λ, we differentiate $r\bar{b} = \Omega(\bar{b})$ to obtain

$$r d\bar{b} = \sigma\theta\left[-S(z)d\Lambda + S(z+\bar{b})d\Lambda + \Lambda S'(z+\bar{b})d\bar{b}\right].$$

When money is not valued,

$$\left.\frac{d\bar{b}}{d\Lambda}\right|_{\phi=0} = \frac{\sigma\theta S(\bar{b})}{r - \sigma\theta\Lambda S'(\bar{b})} > 0$$

for all $\bar{b} \in (0, \bar{b}_1)$ since $r > \sigma\theta\Lambda S'(\bar{b})$ when $\bar{b} > 0$. Similarly when money is valued,

$$\left.\frac{d\bar{b}}{d\Lambda}\right|_{\phi>0} = \frac{\sigma\theta\left[S(z+\bar{b}) - S(z)\right]}{r - \sigma\theta\Lambda S'(z+\bar{b})} > 0$$

for all $\bar{b} \in (0, \bar{b}_0)$ since $S(z+\bar{b}) > S(z)$ and $r > \sigma\theta\Lambda S'(z+\bar{b})$.

Next, we verify that the debt limit is a strictly convex function of Λ for all $\bar{b} < \bar{b}_1$:

$$\left.\frac{d^2\bar{b}}{d\Lambda^2}\right|_{\phi>0} = \frac{(\sigma\theta)^2\left[S(z+\bar{b}) - S(z)\right]S'(z+\bar{b})}{\left[r - \sigma\theta\Lambda S'(z+\bar{b})\right]^2} > 0,$$

since $S'(z+\bar{b}) > 0$ when $\bar{b} < \bar{b}_1$. The derivative when money is not valued is similar but with $z = 0$.

Finally, $\bar{b}$ is linear in Λ when $\bar{b} \geq \bar{b}_1$: $\frac{d\bar{b}}{d\Lambda}|_{\bar{b}\geq\bar{b}_1} > 0$ and $\frac{d^2\bar{b}}{d\Lambda^2}|_{\bar{b}\geq\bar{b}_1} = 0$ since $S'(\bar{b}_1) = 0$. □

Proof of Lemma 5. To prove Lemma 5, we make use of the fact that the seller's benefit of investing is increasing in the measure of sellers that invest. With Kalai bargaining, the seller's benefit of investing is

$$\bar{\kappa}(\bar{b}) = \sigma(1-\theta)[S(z+\bar{b}) - S(z)].$$

Hence, for a given debt limit and choice of real balances by buyers, we have

$$\frac{\partial\bar{\kappa}(\bar{b})}{\partial\Lambda} = \sigma(1-\theta)\left\{S'(z+\bar{b})\frac{d\bar{b}}{d\Lambda} + [S'(z+\bar{b}) - S'(z)]\frac{dz}{d\bar{b}}\right\}.$$

Recall $S(\cdot)$ is increasing and concave, and from Lemma 4, the debt limit is increasing in Λ. If money is valued, then for all $\bar{b} < \bar{b}_0$,

$$\left.\frac{\partial\bar{\kappa}(\bar{b})}{\partial\Lambda}\right|_{\phi>0} = \sigma(1-\theta)\left\{\underbrace{S'(z+\bar{b})}_{(+)}\underbrace{\frac{d\bar{b}}{d\Lambda}}_{(+)} + \underbrace{[S'(z+\bar{b}) - S'(z)]}_{(-)}\underbrace{\frac{dz}{d\bar{b}}}_{(-)}\right\} > 0,$$

Similarly, if money is not valued, then for all $\bar{b} < \bar{b}_1$,

$$\left.\frac{\partial\bar{\kappa}(\bar{b})}{\partial\Lambda}\right|_{\phi=0} = \sigma(1-\theta)\underbrace{S'(\bar{b})}_{(+)}\underbrace{\frac{d\bar{b}}{d\Lambda}}_{(+)} > 0$$

and $\frac{\partial\bar{\kappa}(\bar{b})}{\partial\Lambda}|_{\phi=0} = 0$ otherwise. Hence, for a given debt limit and choice of real balances by buyers, the seller's benefit of investing is increasing in Λ.

We now turn to the proof of Lemma 5 proper. In the following, we consider the case where $i > \hat{i}$; the case where $i < \hat{i}$ is similar except the threshold $\bar{b}_0$ does not exist and should be replaced with $\bar{b}_1 - \bar{z}$.

First, notice when $\bar{b} = 0$, the seller's expected benefit of investing is zero, whether or not money is valued: $\bar{\kappa}(0)|_{\phi=0} = \bar{\kappa}(0)|_{\phi>0} = 0$. Consequently, no sellers will invest: $\Lambda = 0$. For $\bar{b} > 0$, we now consider separately the cases when (*1*) money is not valued ($\phi = 0$) and (*2*) money is valued ($\phi > 0$). We also show (*3*) $\Lambda(\bar{b})|_{\phi=0} > \Lambda(\bar{b})|_{\phi>0}$ for a given $\bar{b} < \bar{b}_0$, and $\Lambda(\bar{b})|_{\phi=0} \to \Lambda(\bar{b})|_{\phi>0}$ as $\bar{b} \to \bar{b}_0$.

(1) When money is not valued, the seller's expected benefit of investing when $\bar{b} \in (0, \bar{b}_1)$ is a strictly increasing and concave function of the debt limit:

$$\bar{\kappa}'(\bar{b})|_{\phi=0} = \sigma(1-\theta)S'(\bar{b}) > 0,$$
$$\bar{\kappa}''(\bar{b})|_{\phi=0} = \sigma(1-\theta)S''(\bar{b}) < 0,$$

since $S'(\bar{b}) > 0$ and $S''(\bar{b}) < 0$ for $\bar{b} \in (0, \bar{b}_1)$.

When $\bar{b} = \bar{b}_1$, buyers can borrow enough to purchase the first-best. In that case, the seller's benefit of investing is at its maximum:

$$\bar{\kappa}_{\max} \equiv \sigma(1-\theta)S^*,$$

where $S^* \equiv u(q^*) - c(q^*)$. Sellers with $\kappa < \bar{\kappa}_{\max}$ will invest, and those with $\kappa > \bar{\kappa}_{\max}$ will not invest. Let Λ_s denote the aggregate measure of sellers that invest when $\bar{b} = \bar{b}_1$. Since $\frac{d\bar{\kappa}_{\max}}{d\bar{b}} = 0$, the aggregate measure of sellers who invest when $\bar{b} > \bar{b}_1$ is also $\Lambda_s \le 1$.

(2) When money is valued, the expected benefit of investing is

$$\bar{\kappa}(\bar{b})|_{\phi>0} = \sigma(1-\theta)[S(z+\bar{b}) - S(z)],$$

which is increasing in debt limit for all $\bar{b} \in (0, \bar{b}_0)$. To verify, differentiate to obtain

$$\bar{\kappa}'(\bar{b})|_{\phi>0} = \sigma(1-\theta)\left\{S'(z+\bar{b}) + \left[S'(z+\bar{b}) - S'(z)\right]\frac{dz}{d\bar{b}}\right\} > 0,$$

where $S'(z+\bar{b}) < S'(z)$ since S is concave and

$$\frac{dz}{d\bar{b}} = -\left[1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''(z+\bar{b})}\right]^{-1} < 0$$

since $S''(z) < 0$ and $S''(z+\bar{b}) < 0$ for all $\bar{b} \in (0, \bar{b}_0)$. Consequently, $\frac{d\bar{\kappa}}{d\bar{b}}|_{\phi>0} > 0$.

(3) For all $\bar{b} \in (0, \bar{b}_0)$, $\bar{\kappa}(\bar{b})|_{\phi=0} > \bar{\kappa}(\bar{b})|_{\phi>0}$ since $S(\bar{b}) > S(z+\bar{b}) - S(z)$ given the concavity of $S(\cdot)$. At $\bar{b} = \bar{b}_0$, we have $z = 0$ and hence $\bar{\kappa}(\bar{b})|_{\phi=0} = \bar{\kappa}(\bar{b})|_{\phi>0}$. □

Proof of Proposition 2. We prove Proposition 2 in two steps.

(1) Given the assumptions made on the cost function, we first show that for a given debt limit and value of money, there exists an equilibrium where Λ sellers invest to access the record-keeping technology. To show that an equilibrium exists, we make use of the fact that the seller's benefit of investing is increasing in the measure of sellers that invest (see the proof of Lemma 5). This immediately implies that it is never the case that a seller invests when a fraction Λ' of other sellers invest but not when $\Lambda'' > \Lambda'$ others invest. Since we assume that κ is zero for a positive measure of sellers and κ is arbitrarily high for a positive measure of sellers, there will always be some sellers that invest and some other sellers who do not. Aggregating across all sellers with respect to $F(\kappa)$ then implies that we can always find an equilibrium where $\Lambda \in (0, 1)$ sellers invest. We cannot show generically there always exists multiple types of a particular equilibrium, e.g. multiple pure credit equilibria with different values for the debt limit $\bar{b}$, but it is possible to construct examples (see e.g. the text).

(2) Once we determine Λ, we know from Section 4 that we can find an equilibrium debt limit. All other endogenous variables can then easily be obtained as before. □

A.2. Welfare in a money-credit equilibrium

Steady-state welfare in a money-credit equilibrium is given by

$$\mathcal{W}^{mc} = \sigma[\Lambda S(z+\bar{b}) + (1-\Lambda)S(z)] - k.$$

To ease computation, we begin by taking $\Lambda \in (0,1)$ as given. Totally differentiating $\mathcal{W}^{mc}$ with respect to i, we obtain:

$$\frac{d\mathcal{W}^{mc}}{di} = \sigma\Lambda S'\left(z+\bar{b}\right)\left[\underbrace{\frac{dz}{di}}_{(-)} + \left(\underbrace{\frac{dz}{d\bar{b}}}_{(-)} + 1\right)\underbrace{\frac{d\bar{b}}{di}}_{(+)}\right] + \sigma\left(1-\Lambda\right)S'\left(z\right)\underbrace{\frac{dz}{di}}_{(-)},$$

where

$$\frac{dz}{d\bar{b}} = -\left[1 + \frac{(1-\Lambda)\,S''(z)}{\Lambda S''\left(z+\bar{b}\right)}\right]^{-1} \in (-1,0),$$

$$\frac{dz}{di} = \left[\sigma\theta\left[(1-\Lambda)\,S''(z) + \Lambda S''\left(z+\bar{b}\right)\right]\right]^{-1} < 0,$$

$$\frac{d\widetilde{z}}{di} = \left[\sigma\theta S''(\widetilde{z})\right]^{-1} < 0,$$

and

$$\frac{d\bar{b}}{di} = \frac{\widetilde{z} - z}{r - \sigma\theta\Lambda S'\left(z+\bar{b}\right)} > 0,$$

since $S''(\cdot) < 0$, $\widetilde{z} > z$ and $r - \sigma\theta\Lambda S'\left(z+\bar{b}\right) > 0$.

Hence, a necessary and sufficient condition for $\frac{d\mathcal{W}^{mc}}{di} < 0$ is

$$\frac{dz}{di} + \left(\frac{dz}{d\bar{b}} + 1\right)\frac{d\bar{b}}{di} < 0$$

Let $\frac{dz}{d\bar{b}} = -a$ where $a \equiv \left[1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''\left(z+\bar{b}\right)}\right]^{-1}$. Consequently,

$$\frac{d\mathcal{W}^{mc}}{di} = \sigma\Lambda S'\left(z+\bar{b}\right)\left[\frac{dz}{di} + (1-a)\frac{d\bar{b}}{di}\right] + \sigma\left(1-\Lambda\right)S'\left(z\right)\frac{dz}{di}$$

and

$$\begin{aligned}\frac{d\mathcal{W}^{mc}}{di} &< 0 \\ &\Longleftrightarrow \frac{dz}{di} + (1-a)\frac{d\bar{b}}{di} < 0 \\ &\Longleftrightarrow \frac{dz}{d\bar{b}} < a-1 \\ &\Longleftrightarrow a > \frac{1}{2}.\end{aligned}$$

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The condition $a > \frac{1}{2}$ is satisfied when $\frac{(1-\Lambda)S''(z)}{\Lambda S''(z+\bar{b})} < 1$ or

$$\Lambda > \frac{S''(z)}{S''(z) + S''\left(z+\bar{b}\right)} \equiv \widehat{\Lambda}.$$

Therefore, welfare in a money-credit equilibrium is decreasing with inflation when $a \in \left(\frac{1}{2}, 1\right)$, or equivalently when $\Lambda > \widehat{\Lambda}$. Notice when Λ is close to 0, $\frac{d\mathcal{W}^{mc}}{di} \approx \sigma S'(z) \frac{dz}{di} < 0$. Finally, when $a < \frac{1}{2}$, or $\Lambda < \widehat{\Lambda}$, we have $\sigma (1-\Lambda) S'(z) \frac{dz}{di} < 0$ and

$$\sigma \Lambda S'\left(z+\bar{b}\right) \left[\frac{dz}{di} + (1-a) \frac{d\bar{b}}{di}\right] > 0$$

In that case, the sign of $\frac{d\mathcal{W}^{mc}}{di}$ is indeterminate.

A.3. Welfare comparison in a money-credit equilibrium vs. pure credit equilibrium

Here we show that $\mathcal{W}^{mc} \geq \mathcal{W}^{c}$ for all $i \in (\underline{i}, \widetilde{i})$ so long as $k > 0$ and the threshold i_c exists, that is if $\mathcal{W}^{m} > \mathcal{W}^{c}$ at $i = 0$ and $\mathcal{W}^{m} < \mathcal{W}^{c}$ at $i = \widetilde{i}$.

First, notice that at $i = 0$, we always have $\mathcal{W}^{m}(0) > \mathcal{W}^{c}(0)$ for any $k > 0$. Next, at $i = \underline{i}$, credit is not feasible given money is valued. Consequently, $\mathcal{W}^{mc}(\underline{i}) = \sigma S(z) = \mathcal{W}^{m}$. Hence at $i = \underline{i}$, $\mathcal{W}^{mc} \geq \mathcal{W}^{c}$. Similarly, at $i = \widetilde{i}$, money is not valued given there is credit. Consequently, $\mathcal{W}^{mc}(\widetilde{i}) = \sigma \Lambda S(\bar{b}) - k = \mathcal{W}^{c}$. Hence at $i = \widetilde{i}$, $\mathcal{W}^{mc} = \mathcal{W}^{c}$. $\square$

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