Estimating labour supply elasticities under rationing: a structural model of time allocation behaviour

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Abstract. This paper presents a structural model of the allocation of time to various non-market activities and market work by couples and single men and women. Parameters are estimated using a sample taken from the UK 2000 Time Use Survey. Own-wage effects are found to be positive for both men and women and are larger for cohabiting individuals than for singles. The presence of young children leads to a much larger increase in the time spent in home production by women than by men. However, the presence of young children causes men to increase their total time spent working by more than women. JEL classification: C34, J22

Calibration des élasticités de l'offre de travail quand il y a rationnement: un modèle structurel de comportement d'allocation du temps. Ce mémoire présente un modèle structurel d'allocation du temps des couples et des célibataires entre le travail sur le marché et diverses activités non-marchandes. Les paramètres sont calibrés à l'aide d'un échantillon tiré du UK 2000 Time Use Survey. On trouve que les effets des salaires propres de chacun sont positifs à la fois pour les hommes et les femmes, et plus importants pour les personnes qui vivent en cohabitation que pour les célibataires. L'existence de jeunes enfants entraîne un accroissement plus grand du temps passé dans la production domestique pour les femmes que pour les hommes. Cependant, l'existence de jeunes enfants entraîne un accroissement du temps total de travail plus important pour les hommes que pour les femmes.

1. Introduction

The disaggregation of leisure into a number of different activities such as sports, media-based activities, home production, and sleep is intuitively appealing: it is

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reasonable to suppose that, for example, it is not the total time spent watching television or playing sports that enters an individual's utility function but rather the time spent doing each of these activities. This conjecture is reflected in several previous studies of time allocation behaviour. For example, Wales and Woodland (1977) divide non-market time into time spent doing housework and pure leisure, Biddle and Hamermesh (1990) differentiate sleep from other forms of non-market time use, while Kooreman and Kapteyn (1987) model the allocation of time by couples between market work and a variety of non-market activities, including home production, child-care, hobbies, and personal care.

This paper presents a structural model of the allocation of time by both couples and single men and women. Household utility is a function of the time allocated by each household member to each of a number of non-market activities and the household's consumption of an aggregate good. We estimate the structural parameters describing households' preferences for time in non-market activities and consumption, recognizing that zero observations for the time allocated to non-market activities or market work imply the presence of constraints on household behaviour. This feature of the model is empirically important, as in many time use surveys, including the one employed in this study, zero observations for the time allocated to market work and many non-market activities are very common. Furthermore, theoretical models of choice under rationing, including Lee and Pitt (1986), Neary and Roberts (1980), and Wales and Woodland (1983), imply that treating the zero observations in the same was as the positive observations leads to inconsistent estimates of quantities of interest. The theoretically consistent treatment of the zero observations adopted in this study represents an innovation relative to the previous literature. Indeed, Kalton (1985) notes that zero observations for the time allocated to particular activities cannot be treated as being representative of the individual's time allocation behaviour, but does not propose a methodology for dealing with such observations. Moreover, in many empirical applications, including Kooreman and Kapteyn (1987), adjustments are made in light of the zero observations for the time spent working, but zero observations for other activities are treated in the same fashion as the positive observations.

Estimation uses a sample taken from the UK 2000 Time Use Survey. Separate models are estimated for single men, single women, and couples. The empirical analysis is conducted assuming that preferences take the Stone-Geary form. This specification of preferences is restrictive, but is chosen because it allows a relatively simple and theoretically consistent empirical implementation of the multivariate time allocation model. Furthermore, goodness of fit analysis shows that the Stone-Geary specification of preferences preforms well in terms of its ability to predict the principal features of the sample.

The estimated structural parameters are used to study the time allocation behaviour of single men and women and also that of couples. Particular attention is paid to the effects of changes in wages and household characteristics on the total time spent working, that is, the sum of time spent in market work and time engaged in home production, and how this quantity varies between couples and singles. While several previous papers have studied the determinants of the total time spent working (see, inter alia, Aguiar and Hurst 2007; Burda, Hamermesh, and Weil 2007; Juster and Stafford 1991; Kalton 1985), the model developed herein provides a more structural and economically grounded perspective on this question. Additionally, the results are compared with those obtained when the large number of zero observations for the time allocated to non-market activities are treated in the same fashion as the positive observations. This comparison is informative about the extent to which conclusions concerning the determinants of labour supply and the time spent in home production are sensitive to the modelling of households' non-market time allocation behaviour.

The results show that single men and single women without children have very similar time allocation behaviour and respond in similar ways to changes in household characteristics and wages. For couples, the time allocation behaviour of the male and female spouses differs substantially, and the effects of individual characteristics, including education and children, on time allocation behaviour differ for men and women. The results further show that treating the zero observations for the time allocated to non-market activities in the same way as the positive observations leads to significant inconsistencies in the estimated effects of individual characteristics and wages on individuals' time allocation behaviour.

This paper proceeds as follows. A model of the allocation of time in the presence of constraints on the time allocated to market work and non-market activities is presented in section 2, while details concerning the empirical implementation and estimation of the model are given in section 3. Section 4 describes the data source and outlines the main features of the time-use behaviour of singles and couples. In section 5 the estimation results are discussed. Section 6 concludes.

2. Modelling the allocation of time under rationing

In this section an economically grounded model of the allocation of time by two adult households between market work and several non-market activities is presented. This model, henceforth referred to as the multivariate time allocation model, forms the basis for the empirical analysis. The corresponding model for single-adult households, which is also used in the empirical work, can be obtained as a special case and therefore is not presented.

The specification of the multivariate time allocation model is as follows. Household i consists of a female spouse and a male spouse. Each spouse's non-market time is disaggregated into J possible uses, denoted by the vectors $T_{if} = (T_{i1f}, \dots, T_{iJf})$ for the female spouse and $T_{im} = (T_{i1m}, \dots, T_{iJm})$ for the male spouse, where T_{ijs} is the time that spouse s living in household i spends in non-market activity j. Each household is assumed to have a well-behaved utility function, $U(T_{if}, T_{im},$

q_i), defined over the time spent by each spouse in each of the J non-market activities and the household's consumption of the aggregate good, q_i.[1]

Household i faces the following optimization problem:

$$\max_{T_{if}, T_{im}, q_i} U(T_{if}, T_{im}, q_i), \tag{1}$$

subject to

$$q_i + w_{if} \sum_{j=1}^{J} T_{ijf} + w_{im} \sum_{j=1}^{J} T_{ijm} \leqslant w_{if} T + w_{im} T + a_i \tag{2a}$$

$$T_{ijf} \geqslant 0, \text{ for } j = 1, \ldots, J \tag{2b}$$

$$T_{ijm} \geqslant 0, \text{ for } j = 1, \ldots, J \tag{2c}$$

$$T - \sum_{j=1}^{J} T_{ijf} \geqslant 0 \tag{2d}$$

$$T - \sum_{j=1}^{J} T_{ijm} \geqslant 0. \tag{2e}$$

In the above, T denotes total available time and $T_{iws} = T - \sum_{j=1}^{J} T_{ijs}$ for $s = f, m$ is the time that spouse s allocates to market work; w_{is} for $s = f, m$ denotes the wage of spouse s, and a_i denotes the household's non-market income. Equation (2a) is the household budget constraint, while equations (2b)–(2c) and (2d)–(2e) detail, respectively, non-negativity constraints on the time spent in non-market activities and market work.[2] Without loss of generality, the price of the aggregate good has been normalized to unity.

The Kuhn-Tucker conditions for this problem are as follows:

$$U_{T_{ijs}} - \lambda_i w_{is} + \mu_{ijs} - \eta_{is} = 0, \text{ for } j = 1, \ldots, J; s = m, f \tag{3a}$$

$$U_{q_i} - \lambda_i = 0 \tag{3b}$$

$$\lambda_i \geqslant 0 \tag{3c}$$

$$\mu_{ijs} \geqslant 0, \text{ for } j = 1, \ldots, J; s = m, f \tag{3d}$$

1 One may interpret the time spent in non-market activities as contributing to the production of commodities that yield utility, as in Becker (1965) and Gronau (1977). In this case, $U(T_{if}, T_{im}, q_i)$ compounds preferences and technology. With sufficiently strong restrictions on preferences over commodities and on the household production technology, the resulting utility function is indeed well behaved (see Pollak and Wachter 1975).

2 The complete problem also includes the constraints $T_{ijs} \leqslant T$ for $j = 1, \ldots, J$ and $s = f, m$, and $T_{iws} \leqslant T$ for $s = f, m$. These constraints are not empirically important and therefore are ignored in what follows.

$$\eta_{is} \geqslant 0, \text{ for } s = m, f, \tag{3e}$$

where λ_i is the multiplier on the budget constraint, μ_{ijs} is the multiplier on the non-negativity constraint for the jth non-market activity for spouse s, and η_{is} is the multiplier on the non-negativity constraint on market work for spouse s. Subscripts on the utility function denote partial derivatives.

Assuming local non-satiation, the budget constraint is strictly binding, implying $\lambda_i > 0$. This allows the first-order conditions given by equation (3a) to be rearranged to produce the following expression:

$$U_{T_{ijs}} - \lambda_i \underbrace{\left(\underbrace{w_{is} + \frac{\eta_{is}}{\lambda_i}}_{w^*_{is}} - \frac{\mu_{ijs}}{\lambda_i} \right)}_{w^*_{ijs}} = 0, \text{ for } j = 1, \ldots, J; s = m, f. \tag{4}$$

Equation (4) implicitly defines w^*_{is}, the reservation wage for spouse s, and w^*_{ijs}, the virtual price of time in non-market activity j for spouse s. Intuitively, an individual's value of time is their wage if they work or their reservation wage if they do not work, and an individual's decision to allocate time to any particular non-market activity depends on their value of time in the non-market activity, relative to their overall value of time.

Solving the Kuhn-Tucker conditions, given by equations (3a)–(3e), produces a system of constrained Marshallian demand functions. If we use the definitions of the reservation wage and the virtual prices of time in the constrained non-market activities, the constrained demand functions can be expressed as the unconstrained demand functions evaluated at the reservation wage and the virtual prices of the constrained non-market activities (for a proof and further discussion see Neary and Roberts 1980). Thus, the demand functions can be written as follows:

$$T^c_{js}(w_{im}, w_{if}, a_i) = T_{js}(w^*_{i1m}, \ldots, w^*_{iJm}, w^*_{i1f}, \ldots, w^*_{iJf}, w^*_{im}, w^*_{if}, a_i), \quad \text{for } j = 1, \ldots, J; s = m, f \tag{5a}$$

$$q^c(w_{im}, w_{if}, a_i) = q(w^*_{i1m}, \ldots, w^*_{iJm}, w^*_{i1f}, \ldots, w^*_{iJf}, w^*_{im}, w^*_{if}, a_i), \tag{5b}$$

where T_{js} and q are *unconstrained* Marshallian demand functions for time in non-market activity j and the aggregate good, and T^c_{js} and q^c denote *constrained* Marshallian demand functions for time in non-market activity j and the aggregate good.

Expressing the demand functions in terms of virtual prices illustrates that a household's demand for time in unconstrained non-market activities depends, through the virtual prices, on the combination of binding and non-binding non-negativity constraints facing the household. Specifically, an observation of zero time allocated to a non-market activity implies a value of time in that activity below the individual's value of time in the activities to which they allocate positive time. This effect, through the virtual price of time in the constrained activity, changes the household's demand for time in the unconstrained activities, relative to the case where the demand for time in the constrained activity is positive. The same is true for labour supply: households facing different combinations of binding and non-binding non-negativity constraints on their non-market time allocations have different labour supply functions. Ignoring any of the corner solutions leads to a misspecified model, and parameter estimates based on such a model will be inconsistent.

3. Empirical specification and estimation

When specifying a functional form for preferences, it is necessary to choose a utility function that permits corner solutions. Also, given that wages are the only observed prices, the demand functions must not involve cross-price effects for a spouse's non-market time uses. In this application, preferences are assumed to take the Stone-Geary form (Stone 1954). While this specification of preferences is restrictive, it has the virtue of allowing a relatively straightforward and theoretically consistent implementation of the multivariate time allocation model. The Stone-Geary utility function takes the following form:

$$U(T_{im}, T_{if}, q_i; \varepsilon_i, Z_i) = \sum_{j=1}^{J} \alpha_{ijm} \log(T_{ijm} - \gamma_{jm}) + \sum_{j=1}^{J} \alpha_{ijf} \log(T_{ijf} - \gamma_{jf}) + \alpha_{iq} \log(q_i - \gamma_q). \tag{6}$$

In the above, Z_i is a vector of observed individual and household characteristics and ε_i represents the unobserved component of household i's preferences. The γ_{js}s can be interpreted as minimum or subsistence quantities. Thus, a corner solution in the time allocated to non-market activity j by spouse s is permitted if γ_{js} is negative.

Maximizing equation (6) subject to the budget constraint described by equation (2a) and ignoring all non-negativity constraints produces the following system of unconstrained Marshallian demand functions for time in non-market activities:

$$T_{ijs} = \gamma_{js} + \frac{\alpha_{ijs}}{w_{is}} \left((w_{im} + w_{if})T + a_i - w_{im} \sum_{j=1}^{J} \gamma_{jm} - w_{if} \sum_{j=1}^{J} \gamma_{jf} - \gamma_q \right), \quad \text{for } j = 1, \ldots, J; s = m, f. \tag{7}$$

As specified, the αs vary across households, while the γs are fixed across individuals.[3] Observed and unobserved individual specific heterogeneity is incorporated in the αs as follows

$$\alpha_{ijs} = \frac{\exp(\varepsilon_{ijs} + Z_i'\beta_{js})}{\sum_{s=m,f}\sum_{j=1}^{J}\exp(\varepsilon_{ijs} + Z_i'\beta_{js}) + \exp(\varepsilon_{iq} + Z_i'\beta_q)},$$
$$\text{for } j = 1, \ldots, J; s = m, f \tag{8a}$$

$$\alpha_{iq} = \frac{\exp(\varepsilon_{iq} + Z_i'\beta_q)}{\sum_{s=m,f}\sum_{j=1}^{J}\exp(\varepsilon_{ijs} + Z_i'\beta_{js}) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}. \tag{8b}$$

When estimating the parameters of the model, the identifying normalizations $\varepsilon_{iJm} = 0$ for all i and $\beta_{Jm} = 0$ are made. Given the above specification of the α_is, the unobserved component of preferences ε_{ijs} represents household i's unobserved preference for time in non-market activity j by spouse s relative to time in the Jth non-market activity by the male spouse. Likewise, $Z_i'\beta_{js}$ represents the observed component of household i's preference for time in non-market activity j by spouse s relative to time in the Jth non-market activity by the male spouse. It is assumed that the ε_{ijs}s are known to the household when it makes its time allocation decision; however, the ε_{ijs}s are unobserved to the econometrician. Furthermore, ε_{ijs} for all i, j, and s is assumed to be independent of Z_i and independent over households.[4]

The model is estimated with assumptions about the distribution of unobserved preferences. Let ε_i denote ε_{ijs} stacked over j and s; ε_i is assumed to be independently and identically normally distributed with zero mean and an unrestricted covariance matrix. The unrestricted covariance matrix allows a flexible pattern of unobserved preferences over time in non-market activities and allows correlations between the spouses' unobserved preferences.

The specification of the α_is given in equations (8a)–(8b) ensures that $0 < \alpha_{ijs} < 1$ for $j = 1, \ldots, J$ and $s = m, f$, $0 < \alpha_{iq} < 1$ and $\sum_{s=m,f}\sum_{j=1}^{J}\alpha_{ijs} + \alpha_{iq} = 1$. The first two of these conditions are necessary and sufficient for global concavity of the cost function and therefore ensure negativity of the demand system. The

3 This specification is not entirely realistic. For example, one might expect that the minimum quantity of goods, γ_q, varies with the number of children in the household. However, given the already complex nature of the model, incorporating variation across individuals in the γs is not attempted.

4 Assuming the unobservables to be independent of Z_i is clearly restrictive: it is plausible to suppose that, for example, individuals with high levels of education have unobserved preferences that are systematically different from the unobserved preferences of individuals with low levels of education. Ignoring any relationship between the explanatory variables and unobserved preferences will lead to inconsistencies. However, the data set used in this application does not contain suitable instrumental variables or proxy variables. Furthermore, given the cross-sectional nature of the data set, there is no intertemporal variation in the explanatory variables. Consequently, panel data techniques are unavailable.

third condition is necessary and sufficient for the demand functions to satisfy adding up and to be homogeneous of degree zero in prices and income. Since the model consists of a system of censored demand functions, it is important to ensure that the model is coherent (see Gourieroux, Laffont, and Monfort 1980; Ransom 1987; van Soest, Kapteyn, and Kooreman 1993). For the model in hand, coherency requires that each realization of the random vector ε_i corresponds to a unique vector of endogenous variables (T_i, q_i), and for every observed (T_i, q_i) there must exist some ε_i that can generate this outcome. Global concavity of the cost function is sufficient, although not necessary, to ensure that the system of censored demand functions is coherent. The above stochastic specification ensures that the cost function is globally concave; thus, the system of censored demand functions is indeed coherent. This property is desirable because it allows the model to be estimated without needing to restrict the parameter space in order to ensure coherency.

Parameter estimates are obtained using maximum likelihood methods. Given the functional form and the above distributional assumptions, the likelihood function can be derived. Each household falls into one of two cases, depending on the combination of binding and non-binding constraints: (i) both spouses work in the market but there may be binding constraints on the time allocated to some non-market activities; (ii) there are binding non-negativity constraints on the time allocated to market work by one spouse or both spouses and possibly also constraints on the time allocated to non-market activities. The appendix contains derivations of the likelihood contributions for each case.

Households facing multiple binding non-negativity constraints have likelihood contributions that contain multivariate normal distribution functions. Specifically, the dimension of the distribution function which a household contributes to the likelihood function is equal to the number of binding non-negativity constraints facing the household. Except in special, restrictive cases, it is computationally difficult to evaluate numerically multivariate normal distribution functions with more than three dimensions. Thus, in this application, the GHK simulator – due to Börsch-Supan and Hajivassiliou (1993), Geweke, Keane, and Runkle (1994), Hajivassiliou and McFadden (1990), and Keane (1994) – is used to evaluate the multivariate normal distribution functions occurring in the likelihood function.

4. An overview of the data

The sample used in this study is taken from the UK 2000 Time Use Survey. As is the custom for this type of survey, time use data for each individual were collected via two 24-hour time-use diaries, one for a weekday and one for a weekend day. When completing the dairies, individuals were asked to record their activity in each 10-minute interval, and in multi-person households all members of the household were asked to complete their time-use diaries on the same two

TABLE 1
Observed time allocation behaviour of single men and women: average hours per day

	Single women			Single men		
Activity	Weekday	Saturday	Sunday	Weekday	Saturday	Sunday
Volunteer work	0.23	0.58*	0.33	0.17	0.27*	0.26
	(0.14)	(0.27)	(0.20)	(0.10)	(0.12)	(0.13)
Social activities	1.47*	2.51	2.25	1.14*	2.07	2.00
	(0.80)	(0.87)	(0.85)	(0.62)	(0.77)	(0.77)
Sports	0.41	0.62*	0.51*	0.52	1.13*	1.16*
	(0.31)	(0.36)	(0.33)	(0.35)	(0.39)	(0.43)
Home production	2.67*	3.32*	3.17*	1.71*	2.70*	1.98*
	(0.96)	(0.97)	(0.94)	(0.86)	(0.92)	(0.89)
Media activities	2.25	2.60*	3.03*	2.41	3.46*	3.88*
	(0.88)	(0.88)	(0.92)	(0.86)	(0.91)	(0.91)
Other time use	1.67	2.07	1.58	1.80	1.87	1.93
	(0.97)	(0.96)	(0.90)	(0.94)	(0.95)	(0.92)
Sleep	10.23*	10.92	12.21*	9.71*	10.38	11.58*
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
Market work	5.07*	1.39	0.92	6.55*	2.11	1.20
	(0.66)	(0.22)	(0.16)	(0.78)	(0.34)	(0.22)
Total work	7.74*	4.71*	4.09*	8.26*	4.81*	3.18*
	(0.98)	(0.98)	(0.93)	(0.93)	(0.97)	(0.93)
Observations	318	142	178	290	135	152

NOTES: Proportion of non-zero observations shown in parentheses. * Indicates a significant difference between men and women at the 5% level.

days. Additionally, household and individual questionnaires were used to gather information about income, wages, and demographic characteristics.

For the purpose of this study, attention is restricted to individuals aged between 18 and 65 years. Retired individuals and students are also excluded, as are all individuals who reported themselves to be suffering from a long-term illness. Any time-use diaries in which the individual reported that the day was unusual for some reason, such as temporary illness or a holiday, were not used in this study. Separate samples were constructed for single men without children, single women without children, and couples with or without children, referred to as cohabiting individuals and defined to include both married and unmarried couples.

Eight different time uses are distinguished: volunteer work; social activities; sports; home production, including child care; media activities; other time use; sleep; and market work. Tables 1 and 2 summarize the observed time allocation behaviour of, respectively, single men and women and couples. For singles, the time allocations patters of men and women are remarkably similar. On weekdays and on weekend days women spend significantly longer in home production than men, while on weekdays women spend significantly less time in market work than men; however, these differences are not particularly large. The total time spent working, defined as market work plus home production, by single men and single

TABLE 2
Observed time allocation behaviour of cohabiting men and women: average hours per day

	Cohabiting women			Cohabiting men		
Activity	Weekday	Saturday	Sunday	Weekday	Saturday	Sunday
Volunteer work	0.23*	0.30	0.34	0.14*	0.29	0.32
	(0.16)	(0.16)	(0.20)	(0.09)	(0.13)	(0.19)
Social activities	1.06*	2.10	1.47	0.74*	1.99	1.34
	(0.73)	(0.82)	(0.81)	(0.58)	(0.74)	(0.73)
Sports	0.30*	0.45*	0.50*	0.49*	0.76*	1.00*
	(0.25)	(0.29)	(0.32)	(0.30)	(0.35)	(0.44)
Home production	4.32*	4.80*	5.03*	1.86*	3.12*	3.46*
	(0.99)	(0.98)	(0.99)	(0.88)	(0.92)	(0.93)
Media activities	2.06*	2.34*	2.77*	2.34*	3.13*	3.28*
	(0.90)	(0.87)	(0.94)	(0.90)	(0.93)	(0.93)
Other time use	1.78	1.77	1.42*	1.73	1.82	1.62*
	(0.96)	(0.94)	(0.86)	(0.96)	(0.95)	(0.91)
Sleep	10.15*	10.98*	11.72*	9.57*	10.62*	11.46*
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
Market work	4.09*	1.27*	0.76*	7.11*	2.26*	1.52*
	(0.62)	(0.21)	(0.17)	(0.83)	(0.34)	(0.25)
Total work	8.41*	6.07*	5.79*	8.97*	5.38*	4.98*
	(1.00)	(0.99)	(1.00)	(0.94)	(0.97)	(0.97)
Observations	1224	632	589	1224	632	589

NOTES: Proportion of non-zero observations shown in parentheses. * Indicates a significant difference between men and women at the 5% level.

women is also similar, although again significant differences exist: on weekdays, men spend around a half-hour per day longer on total work than women, while women work for about an hour longer than men on Sundays. For single men and women, sleep, home production, and media activities are the most common forms of non-market time use. For couples, the time allocation patterns of men and women differ substantially. In particular, on weekdays men spend around three hours per day longer in market work than women. However, women in couples spend significantly longer than men in home production activities both on weekdays and on weekend days. Despite these differences, and in line with the findings of Burda, Hamermesh, and Weil (2007), the total time spent working is similar for men and women; however, as for single individuals, significant differences still remain. Cohabiting women allocate significantly more time to sleep and significantly less time to media activities than cohabiting men.

Observations of zero time allocated to volunteer work, sports, social activities, media activities, home production, and market work are common for single and cohabiting individuals. Thus, when we estimate the multivariate time allocation model, corner solutions for these activities are incorporated. In the empirical implementation, behaviour is conditioned on individual and, in the case of couples, household characteristics. Specifically, for single men and women, controls for age, education – which is defined as either low, medium, or high – and the availability of a car are included.[5] For couples, time allocation behaviour is conditioned on the age and educational attainment of both spouses, the number of children aged 0–4 years and the number of children aged 5–16 years present in the household, and the availability of a car. Additionally, for both singles and couples, indicator variables for Saturdays and Sundays are included. The estimation procedure requires data on household consumption, non-labour income, and wages. Consumption is defined as average daily household consumption, and wages are defined as usual monthly net earnings divided by usual hours of work per month. Consumption and wages are denominated in British pounds. Given consumption, wages, and observed hours of market work on a particular day, non-labour income for that day is inferred from the household budget constraint.

Wages are observed only for individuals who are currently employed. However, in order to estimate the parameters of the multivariate time allocation model, wages are also required for non-working individuals. Thus, the results of an ordinary least squares regression of the log wages of the working individuals on a variety of individual characteristics are used to predict the missing wages. Specifically, for each non-working individual, the log wage is imputed as the conditional mean of the individual's wage as implied by the regression results, scaled such that the variance of the predicted log wages is equal to the variance of the observed log wages for the working individuals. The parameters of the wage equations, estimated separately for men and women, are presented in table 3. Ideally, the parameters of the wage equations would be estimated jointly with the other parameters of the model. However, experimentation with such a model revealed it to be numerically unstable. The simplified model implemented here will yield consistent parameter estimates, provided that the unobserved heterogeneity in the wage equation is independent of the unobserved heterogeneity affecting time allocation decisions. In the case of women, this assumption was tested by estimating a Heckman selection model, where children were included in the selection equation but excluded from the wage equation. The correlation between the unobservables in the selection equation and the unobservables affecting wages was found to be positive but not significantly different from zero.[6]

At this point, it is appropriate to consider the interpretation of the multivariate time allocation model when it is estimated with the above-described time use data. As the time use data refer to the period of a single day, the optimization problem described in section 3 must also refer to one particular day. Given this interpretation, the model implies that a household's observed time allocation on a particular day depends on observed characteristics, wages, and unobserved

5 Medium education is defined as O levels or GCSEs above grade A-C, which are obtained by about 60% of students at age 16 years. High education is defined as A levels or above and is obtained by around 30% of individuals.

6 This exercise was conducted only for women; as is the case for men, there is no obvious variable to exclude from the wage equation. Results of this estimation are contained in the on-line appendix linked to this article at the CJE journal archive http://economics.ca/cje/en/archive.php.

TABLE 3
Wage equations for women and men (singles and couples are pooled). Standard errors are robust to hetroscedasticity

	Women		Men	
	Coefficient	Standard error	Coefficient	Standard error
Age	0.023	0.011	0.041	0.012
$Age^2/100$	−0.025	0.014	−0.043	0.015
High educ.	0.185	0.049	0.150	0.055
Medium educ.	0.033	0.044	0.115	0.048
Upper non-manual	0.562	0.073	0.428	0.061
Lower non-manual	0.341	0.049	0.177	0.053
Skilled manual	0.291	0.043	0.202	0.064
White	−0.013	0.109	0.070	0.111
Unemployment rate	0.003	0.005	−0.007	0.006
North	0.050	0.089	0.001	0.097
Midlands	0.092	0.091	0.069	0.100
Eastern	0.178	0.099	0.068	0.107
Southeast	0.157	0.090	0.022	0.098
Southwest	−0.026	0.097	−0.074	0.108
Scotland	0.091	0.098	0.056	0.106
Intercept	0.806	0.264	0.673	0.281
R^2	11.3%		11.7%	
Observations	1173		11.50	

preferences. On this day, the household may be constrained in one or more aspects of its time allocation behaviour. Variation in the household's time allocation behaviour over time is due to intertemporal variation in unobserved preferences and systematic differences in preferences between weekdays and Saturdays and Sundays. It should be noted that, although the time allocation model refers to one particular day, the above definition of consumption recognizes that households generally smooth their consumption over time and thus do not consume more on days when they work more. Furthermore, the daily nature of the model and the data are taken into account when determining the effects of changes in wages or household characteristics. Specifically, instead of looking at the effect on time allocation behaviour on a particular day, marginal effects are defined as the change in the average time per day allocated to each activity caused by a change in a household characteristic or a wage change.[7]

5. Results

5.1. The multivariate time allocation model with rationing

Parameter estimates for the multivariate time allocation model, estimated separately for single men, single women, and couples, are presented in the on-line

7 In more detail, the average time per day allocated to each activity by a particular individual is computed by taking expectations with respect to the unobservables.

appendix, together with the results of simulations that show that the model is able to predict accurately the principal features of the time allocation patterns of single and cohabiting men and women.[8] In particular, simulations based on the estimated parameters show that the model generates reasonable predictions of the hours per day allocated to each activity and the distribution of zero observations.

The parameter estimates are not discussed in detail, but the following features of the estimates are noted. For couples and single men, the estimated values of the γ_{js}s are negative for all non-market activities except sleep, while for single women the γ_{js}s are negative for all non-market activities, including sleep. For couples, the estimated value of γ_q is 3.43, which corresponds to a subsistence level of consumption of around £24 per week. While this may appear implausibly low, it is important to realize that, given prevailing prices and preferences, all households may choose to consume well in excess of their subsistence consumption. Indeed, for single men and women the estimated subsistence consumption is negative, which again does not imply a negative value of actual consumption. In terms of the unobservables, the largest levels of unobserved preference variation occur for consumption and time spent sleeping, and there are significant correlations between the unobserved preferences for different time usages.

The effects of wages and individual characteristics on households' time allocation behaviour are examined by translating the parameter estimates into marginal effects, reported in table 4 for singles and in table 5 for couples. The figures reported in these tables are averages over the sampled individuals and therefore can be interpreted as the expected change in time allocation behaviour for the sample members. The following discussion focuses on the effects of age, educational qualifications, wages, and, for couples, children on the time allocated to market work and home production, while the allocation of time to other activities is discussed in less detail.

For singles, the effect of age on time allocation behaviour is similar for men and women; the time spent in home production is increasing in age, the time spent in market work is decreasing in age, and the total time spent working is increasing in age. Specifically, a 10-year increase in age leads to an increase in total time spent working of 1.01 hours per week for single women and 1.11 hours per week for single men. Men and women with either high or medium levels of education spend less time in home production than individuals with low educational qualifications. However, the effects of education on the time spent in market work are somewhat different for men and women. In particular, women with high or medium levels of education spend significantly more time in market work than women with low levels of education, while educational attainment has no significant effect on the time allocated to market work by men. For both single men and single women educational qualifications do not significantly affect the total time spent working.

8 Tables are available in an on-line appendix linked to this article at the CJE journal archive, http://economics.ca/cje/en/archive.php.

TABLE 4
Marginal effects for single men and women of changes in individual characteristics on average hours per week allocated to each time use by the sample members. Standard errors in parenthesis

	Vol. work	Social act.	Sports	Home prod.	Media act.	Other	Sleep	Market work	Total work
					Single women				
Age	0.88	−0.64	0.23	3.54	0.56	−0.31	−1.72	−2.54	1.01
	(0.32)	(0.53)	(0.17)	(0.53)	(0.55)	(0.30)	(0.48)	(0.76)	(0.84)
High educ.	−0.71	1.35	1.18	−3.85	−3.55	1.50	−2.24	6.33	2.48
	(0.47)	(1.23)	(0.72)	(1.19)	(1.64)	(0.99)	(1.84)	(3.73)	(3.23)
Med. educ.	0.35	−1.07	0.01	−2.08	−1.77	−0.57	−0.52	5.65	3.58
	(0.98)	(1.32)	(0.58)	(1.48)	(1.60)	(1.26)	(2.21)	(2.70)	(2.37)
Wage	−0.02	−0.02	−0.02	−0.02	−0.04	−0.02	−0.06	0.20	0.18
	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.04)	(0.09)	(0.08)
Average hours per week	2.33	11.74	3.11	19.25	16.27	12.55	73.67	29.09	48.34
	(0.46)	(0.45)	(0.29)	(0.66)	(0.70)	(0.60)	(1.17)	(1.55)	(1.25)
					Single men				
Age	0.44	−0.21	−0.28	3.17	0.53	−0.03	−1.56	−2.06	1.11
	(0.38)	(0.62)	(0.28)	(0.51)	(0.77)	(0.41)	(0.60)	(1.12)	(1.10)
High educ.	0.58	0.22	2.78	−2.42	−5.36	2.77	−4.60	6.02	3.61
	(0.56)	(1.23)	(1.22)	(0.80)	(1.73)	(1.08)	(2.17)	(4.45)	(4.06)
Med. educ.	−0.19	−0.67	2.48	−2.98	−0.91	2.20	−0.40	0.49	−2.49
	(1.55)	(1.35)	(0.99)	(1.21)	(1.84)	(1.45)	(2.61)	(4.14)	(3.47)
Wage	−0.03	−0.03	−0.04	−0.01	−0.06	−0.01	−0.07	0.25	0.24
	(0.03)	(0.02)	(0.01)	(0.01)	(0.03)	(0.01)	(0.04)	(0.11)	(0.09)
Average hours per week	1.63	9.68	5.39	12.35	18.64	12.62	69.50	38.19	50.54
	(1.51)	(0.75)	(0.42)	(0.57)	(1.10)	(0.59)	(1.46)	(2.33)	(2.13)

NOTES: Marginal effect of age is the effect of a 10-year increase in age. Marginal effect of medium education is the effect of an increase in education from low education to medium education, while the marginal effect of high education is the effect of an increase from low education to high education. Wage effects refer to the effect of a 10% increase in wages.

TABLE 5
Marginal effects for cohabiting men and women of changes in individual and household characteristics on average hours per week allocated to each time use by the sample members. Standard errors in parenthesis

	Cohabiting females									Cohabiting males								
	Vol. work	Social act.	Sports	Home prod.	Media act.	Other	Sleep	Market work	Total work	Vol. work	Social act.	Sports	Home prod.	Media act.	Other	Sleep	Market work	Total work
Female's age	0.53	−1.00	0.33	1.48	0.60	−1.36	−0.36	−0.22	1.27	0.34	−0.95	0.45	−0.58	0.52	−0.30	−0.56	1.08	0.49
	(0.24)	(0.59)	(0.30)	(0.52)	(0.81)	(0.56)	(0.70)	(1.40)	(1.42)	(0.24)	(0.48)	(0.33)	(0.47)	(0.84)	(0.62)	(0.45)	(1.50)	(1.18)
Female high educ.	−0.10	0.86	1.06	−0.89	−2.74	2.16	−0.50	0.14	−0.75	−0.06	0.06	0.37	−0.21	−2.55	1.34	−0.42	1.46	1.25
	(0.13)	(0.51)	(0.27)	(0.78)	(0.61)	(0.55)	(0.79)	(1.59)	(1.19)	(0.25)	(0.39)	(0.27)	(0.45)	(0.69)	(0.76)	(0.93)	(1.96)	(1.81)
Female med. educ.	0.01	0.06	0.48	−0.13	−1.42	0.61	0.11	0.28	0.15	−0.01	0.12	0.09	−0.32	−0.56	0.83	0.35	−0.49	−0.82
	(0.18)	(0.39)	(0.30)	(0.91)	(0.73)	(0.65)	(0.71)	(1.51)	(1.59)	(0.48)	(0.27)	(0.28)	(0.58)	(0.98)	(0.62)	(0.89)	(1.71)	(1.45)
Male's age	0.02	1.11	−0.10	0.93	−0.23	0.75	−1.41	−1.07	−0.14	−0.09	1.21	−0.27	1.20	0.29	−0.06	−0.70	−1.57	−0.37
	(0.18)	(0.78)	(0.32)	(0.56)	(0.74)	(0.42)	(0.63)	(1.56)	(1.56)	(0.10)	(0.68)	(0.17)	(0.54)	(0.85)	(0.56)	(0.61)	(1.71)	(1.42)
Male high educ.	0.34	−0.07	0.16	−2.88	−0.82	−0.23	−1.42	4.91	2.03	0.36	1.24	1.61	1.24	−0.46	1.58	−0.85	−4.72	−3.48
	(0.23)	(0.40)	(0.29)	(0.77)	(0.60)	(0.57)	(0.70)	(2.01)	(1.63)	(0.38)	(0.37)	(0.45)	(0.56)	(0.68)	(0.44)	(1.25)	(2.45)	(2.35)
Male med educ.	−0.11	0.38	0.28	−1.41	−0.02	−0.76	−1.06	2.70	1.29	0.06	0.56	0.69	1.45	0.30	0.23	−0.87	−2.42	−0.96
	(0.11)	(0.54)	(0.31)	(0.71)	(0.47)	(0.54)	(0.59)	(0.81)	(0.61)	(0.38)	(0.37)	(0.32)	(0.52)	(0.87)	(0.34)	(0.93)	(1.41)	(1.46)
Young child	0.14	0.25	−0.15	12.20	0.40	0.29	−1.16	−11.97	0.23	−0.19	−0.46	−0.63	1.99	−2.82	−0.50	−1.99	4.60	6.59
	(0.14)	(0.36)	(0.17)	(0.76)	(0.42)	(0.32)	(0.67)	(1.38)	(0.92)	(0.60)	(0.24)	(0.34)	(0.67)	(0.63)	(0.43)	(0.96)	(1.74)	(1.44)
Old child	0.09	−0.24	−0.11	2.68	−0.69	0.53	−0.80	−1.46	1.22	0.11	0.14	0.21	0.60	−0.04	0.63	−0.02	−1.63	−1.03
	(0.06)	(0.41)	(0.12)	(0.35)	(0.26)	(0.30)	(0.31)	(0.78)	(0.74)	(0.15)	(0.13)	(0.16)	(0.25)	(0.26)	(0.23)	(0.30)	(0.52)	(0.47)
Female's wage	−0.04	−0.16	−0.09	−0.50	−0.29	−0.18	−0.51	1.77	1.28	0.03	0.18	0.11	0.29	0.45	0.24	0.68	−1.98	−1.68
	(0.05)	(0.04)	(0.05)	(0.06)	(0.08)	(0.06)	(0.08)	(0.10)	(0.12)	(0.05)	(0.04)	(0.03)	(0.04)	(0.05)	(0.02)	(0.05)	(0.08)	(0.07)
Male's wage	0.03	0.17	0.08	0.60	0.37	0.20	0.61	−2 0 .05	−1.45	−0.04	−0.12	−0.07	−0.14	−0.18	−0.18	−0.55	1.28	1.14
	(0.01)	(0.03)	(0.02)	(0.05)	(0.03)	(0.04)	(0.06)	(0.08)	(0.09)	(0.04)	(0.04)	(0.03)	(0.07)	(0.06)	(0.08)	(0.07)	(0.11)	(0.13)
Average hours per week	1.06	8.47	2.64	28.47	14.68	12.18	79.25	21.24	49.72	1.16	6.69	3.40	13.14	18.15	12.98	72.83	39.66	52.80
	(0.17)	(0.33)	(0.22)	(0.96)	(0.39)	(0.27)	(1.61)	(1.03)	(1.16)	(1.67)	(0.78)	(0.63)	(0.32)	(0.51)	(0.39)	(0.75)	(1.37)	(1.33)

NOTES: The marginal effect of a young child is the effect of change from no children to having one young child, while the marginal effect of an old child is the effect of a change from no children to one old child. Also see notes for table 4.

For both single men and single women, a 10% increase in wages leads to an increase in market work of around a quarter-hour per week. This increase in the time spent working is compensated for by a reduction in the time allocated to all non-market activities, especially sleep. Following the wage increase, the time allocated to home production decreases by only a small amount, and therefore, for both single men and single women, the total time spent working is increasing in the wage.

In contrast to the results for single men and women, for couples there are significant differences between men and women in the effects of individual characteristics on time allocation behaviour. For cohabiting women, the time spent in home production is significantly increasing in age, while for cohabiting men age has no significant effect on the time allocated to home production. For cohabiting men and women, age does not significantly affect the time allocated to either market work or total work. The educational attainment of the female spouse does not have a significant effect on the time allocation behaviour of either spouse. In contrast, an increase in the male's educational qualifications significantly increases the time that he spends in home production and leads to a significant decrease in the time that the female spouse allocates to home production. This effect on the female spouse is large: an increase in the male spouses's education from low to high reduces the time spent by the female spouse in home production by almost three hours per week while an increase from low education to medium education reduces the female's time spent in home production by around one and a half hours per week. Given that on average women allocate approximately 15 hours per week longer to home production than men, this implies that higher educational qualifications for the man lead to a more equitable allocation of home production activities within the household. Overall, the time spent in market work and the total time spent working by the female spouse are higher for women whose spouses have medium or high qualifications than for women whose spouses have low qualifications, while higher education qualifications decrease the man's time spent in market work and also his total time spent working.

The time spent by the female spouse in home production is highly sensitive to the presence of young children. Specifically, the presence of a young child in the household increases the time spent by the female spouse in home production by 12.20 hours per week. However, this effect is matched almost exactly by a reduction in the time spent by the female spouse in market work; the total time spent working and the time spent in other non-market activities are not significantly affected by the presence of a young child in the household. Men's time-use behaviour is affected differently by the presence of young children in the household. A young child increase the time spent by the male spouse in home production by about two hours per week. However, in contrast to women, men increase the time spent in market work in response to an increase in the number of young children in the household, and reduce the time spent sleeping or engaged in media activities. Thus, while women do not alter the total amount of time spent working

TABLE 6
Comparison of marginal effects for single men and women based on a multivariate time allocation models allowing and ignoring rationing for the time allocated to non-market activities

	Home production			Market work		
	Allowing rationing	Ignoring rationing	2-sided p-value for difference	Allowing rationing	Ignoring rationing	2-sided p-value for difference
			Single women			
Age	3.54	3.29	0.410	−2.54	−2.67	0.615
High educ.	−3.85	−4.74	0.041	6.33	4.31	0.032
Med educ.	−2.08	−2.29	0.646	5.65	4.85	0.272
Wage	−0.0203	−0.0281	0.210	0.2009	0.1886	0.685
			Single men			
Age	3.17	3.13	0.831	−1.69	−2.06	0.153
High educ.	−2.42	−2.24	0.494	3.38	6.02	0.003
Med educ.	−2.98	−2.60	0.301	−0.06	0.49	0.432
Wage	−0.0097	−0.0028	0.06	0.1565	0.2484	0.001

NOTES: Figures are average hours per week for the sample members. Also see the notes for table 4.

in response to having young children, men work longer at home and also spend more time in market work.

Own wage effects are positive for both men and women. Specifically, a 10% increase in the females' wages increases female labour supply by an average of 1.77 hours per week, which corresponds to a wage elasticity of average hours of market work of 0.80. For men, the own-wage effect is smaller: a 10% increase in the males' wages leads to an increase of 1.28 hours per week in average hours, corresponding to a wage elasticity of average hours of 0.32. When increasing hours of work, both men and women tend to reduce the time spent sleeping, but women tend to reduce time spent in home production activities more than men. Thus, home production activities are a closer substitute for market work for women than for men. Examination of the cross-wage effect reveals significant negative effects for both men and women.

5.2. The importance of rationing

As explained above, ignoring the presence of constraints on households' non-market time allocation behaviour will lead to inconsistent estimates of marginal effects and other quantities of interest. Tables 6 and 7 explore the nature of the inconsistencies arising from neglecting corner solutions for the time allocated to non-market activities. Specifically, these tables compare the marginal effects implied by the above multivariate time allocation model, which allows rationing of the time allocated to several non-market activities and market work, with the marginal effects obtained from a version of the above model that ignores any

TABLE 7
Comparison of marginal effects for cohabiting men and women based on a multivariate time allocation models allowing and ignoring rationing for the time allocated to non-market activities

	Home production			Market work		
	Allowing rationing	Ignoring rationing	2-sided p-value for difference	Allowing rationing	Ignoring rationing	2-sided p-value for difference
			Cohabiting females			
Female's age	1.48	1.00	0.003	−0.22	0.21	0.112
Female high educ.	−0.89	−1.66	0.002	0.14	0.81	0.165
Female med. educ.	−0.13	0.79	0.215	0.28	0.61	0.443
Male's age	0.93	0.42	0.041	−1.07	−0.09	0.000
Male high educ.	−2.88	−3.08	0.242	4.91	4.46	0.671
Male med. educ.	−1.41	−1.58	0.645	2.70	2.52	0.771
Young child	12.20	10.13	0.001	−11.97	−8.41	0.000
Old child	2.68	2.22	0.002	−1.46	−1.30	0.347
Female's wage	−0.50	−0.38	0.087	1.77	0.78	0.000
Male's wage	0.60	0.44	0.147	−2.05	−1.23	0.000
			Cohabiting males			
Female's age	−0.58	−0.30	0.267	1.08	1.31	0.617
Female high educ.	−0.21	0.29	0.675	1.46	0.40	0.193
Female med. educ.	−0.32	−0.02	0.689	−0.49	−0.21	0.503
Male's age	1.20	1.71	0.056	−1.57	−2.27	0.080
Male high educ.	1.24	1.04	0.617	−4.72	−2.79	0.042
Male med. educ.	1.45	1.67	0.490	−2.42	−1.84	0.395
Young child	1.99	3.50	0.002	4.60	1.98	0.000
Old child	0.60	0.69	0.681	−1.63	−0.70	0.001
Female's wage	0.29	0.12	0.034	−1.98	−1.12	0.000
Male's wage	−0.14	−0.08	0.645	1.28	0.68	0.000

NOTES: Figures are average hours per week for the sample members. Also see the notes for table 5.

rationing of the time allocated to non-market activities. In the latter model, zero observations for the time allocated to non-market activities are treated in the same way as the positive observations, while zero observations for the time allocated to market work are assumed to correspond to binding non-negativity constraint on individuals' labour supply behaviour. When we compare the marginal effects, attention is restricted to the marginal effects for the time spent in market work and the time allocated to home production.

For both single and cohabiting men and women, the marginal effects of individual characteristics on the time allocated to market work differ significantly between the two models. For example, for single women the estimated effect of an increase in her educational qualifications from low to high on the hours per weeks spent in market work is 6.33 when rationing is allowed and 4.31 when rationing is ignored. There are also significant differences in the own-wage effects for market work. For example, for cohabiting women a 10% increase in the woman's wage raises the hours per week spent in market work by 1.77 when rationing is included and 0.78 hours per week when rationing is ignored. The differences between the predictions of the two models in terms of the determinants of the time allocated to home production are not as large, but some are significant. For both market work and home production, the biases introduced by ignoring rationing of the time allocated to non-market activities are not systematically in one direction. In total, these results imply that conclusions concerning labour supply and, to a lesser extent, the time allocated to home production are sensitive to the approach taken to modelling the time allocated to non-market activities.

6. Conclusion

This paper has developed a model of households' allocation of time between market work and several non-market activities that incorporates an economically grounded mechanism for generating the large number of zero observations for the time allocated to non-market activities. As suggested by theoretical models of choice under rationing, neglecting the corner solutions for the time allocated to non-market activities has been found to produce significant inconsistencies in the estimated effects of individual characteristics on the time allocation behaviour of single and cohabiting men and women. Furthermore, for cohabiting men and women own- and cross-wage effects are significantly larger in magnitude when rationing is permitted than when rationing is ignored, while the reverse is true for single men.

The results regarding the time allocation behaviour of cohabiting individuals reveal some non-obvious patterns of the division of activities within the household. Interestingly, market work and home production are closer substitutes for women than for men. Furthermore, it has been found that women do not disproportionately bear the burden of child care, once adjustments in market work are also considered.

Appendix: Likelihood contributions

We first consider the case where the non-negativity constraints on the time spent in the first l non-market activities by the male spouse are binding, and all other constraints are non-binding. Similar expressions apply when the female spouse also faces binding constraints on her non-market time allocation behaviour. The first-order conditions are as follows:

$$\frac{\alpha_{ijm}}{-\gamma_{jm}} - \lambda_i w^*_{ijm} = 0, \text{ for } j = 1, \ldots, l \tag{A1a}$$

$$\frac{\alpha_{ijm}}{T_{ijm} - \gamma_{jm}} - \lambda_i w_{im} = 0, \text{ for } j = l + 1, \ldots, J \tag{A1b}$$

$$\frac{\alpha_{ijf}}{T_{ijf} - \gamma_{jf}} - \lambda_i w_{if} = 0, \text{ for } j = 1, \ldots, J \tag{A1c}$$

$$\frac{\alpha_{iq}}{q_i - \gamma_q} - \lambda_i = 0. \tag{A1d}$$

Assume, without loss of generality, that both spouses allocate positive time to the Jth non-market activity. Manipulation of the first-order conditions yields

$$P(w_{ijm} \leqslant w_{im} | Z_i) = P(\varepsilon_{ijm} \leqslant \log(-\gamma_{jm}) - \log(T_{iJm} - \gamma_{Jm}) - Z_i' \beta_{jm}),$$
$$\text{for } j = 1, \ldots, l \tag{A2a}$$

$$\varepsilon_{ijm} = \log(T_{ijm} - \gamma_{jm}) - \log(T_{iJm} - \gamma_{Jm}) - Z_i' \beta_{jm},$$
$$\text{for } j = l + 1, \ldots, J - 1 \tag{A2b}$$

$$\varepsilon_{ijf} - \varepsilon_{iJf} = \log(T_{ijf} - \gamma_{jf}) - \log(T_{iJf} - \gamma_{Jf}) - Z_i'(\beta_{jf} - \beta_{Jf}),$$
$$\text{for } j = 1, \ldots, J - 1 \tag{A2c}$$

$$\varepsilon_{iq} - \varepsilon_{iJf} = \log(q_i - \gamma_q) - \log(T_{iJf} - \gamma_{Jf}) - \log(w_{if}) - Z_i'(\beta_q - \beta_{Jf}) \tag{A2d}$$

$$\varepsilon_{iq} = \log(q_i - \gamma_q) - \log(T_{iJm} - \gamma_{Jm}) - \log(w_{im}) - Z_i' \beta_q. \tag{A2e}$$

Thus, the household's contribution to the likelihood is as follows:

$$\begin{aligned} L_{i2} = {} & P(w^*_{i1m} \leqslant w_{im}, \ldots, w^*_{ilm} \leqslant w_{im} \mid T_{il+1m}, \ldots, \\ & T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i, w_{im}, w_{if}, Z_i) \\ & \times f_2(T_{il+1m}, \ldots, T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i \mid Z_i, w_{if}, w_{im}) \end{aligned} \tag{A3a}$$

$$\begin{aligned} = {} & P(w^*_{i1m} \leqslant w_{im}, \ldots, w^*_{ilm} \leqslant w_{im} \mid T_{il+1m}, \ldots, \\ & T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i, w_{im}, w_{if}, Z_i) \\ & \times f_{2a}(\varepsilon_{il+1m}, \ldots \varepsilon_{iJ-1m}, \varepsilon_{i1f}, \ldots, \varepsilon_{iJf}, \varepsilon_{iq} \mid w_{im}, w_{if}, Z_i) \left| \frac{\partial \check{\varepsilon}_i}{\partial \check{T}_i} \right|, \end{aligned} \tag{A3b}$$

where $P(.)$ is an l variate normal distribution function; f_2 is the joint density of $\check{T}_i = (T_{il+1m}, \ldots, T_{iJ-1m}, T_{ilf}, \ldots, T_{iJ-1m}, q_i)$, conditional on the regressors Z_i; f_{2a} is the multivariate normal density function of $\check{\varepsilon}_i = (\varepsilon_{il+1m}, \ldots \varepsilon_{iJ-1m}, \varepsilon_{i1f}, \ldots \varepsilon_{iJf}, \varepsilon_{iq})$; and $|\partial \check{\varepsilon}_i / \partial \check{T}_i|$ is the absolute Jacobian from

$\check{T}_i$ to $\check{\varepsilon}_i$. Using the budget constraint shown in equation (2a) and the utility function given in equation (6) gives the following:

$$\left|\frac{\partial \check{\varepsilon}_i}{\partial \check{T}_i}\right| = \frac{\sum_{j=l+1}^{J}(T_{ijm}-\gamma_{jm}) + \dfrac{q_i-\gamma_q}{w_{im}} + \sum_{j=1}^{J}(T_{ijf}-\gamma_{jf}) + \dfrac{q_i-\gamma_q}{w_{if}}}{\left(\dfrac{q_i-\gamma_q}{w_{im}}\right)\prod_{j=l+1}^{J}(T_{ijm}-\gamma_j m)\left(\dfrac{q_i-\gamma_q}{w_{im}}\right)\prod_{j=1}^{J}(T_{ijm}-\gamma_{jm})}, \quad \text{(A4)}$$

where the simplification of the absolute Jacobian follows Graybill (1983).

Attention is now turned to the case where the non-negativity constraints on the time spent in the first l non-market activities by the male spouse are binding and the male spouse does not work. All other constraints are non-binding. The first-order conditions are given by

$$\frac{\alpha_{ijm}}{-\gamma_{jm}} - \lambda_i w^*_{ijm} = 0, \text{ for } j = 1, \ldots, l \quad \text{(A5a)}$$

$$\frac{\alpha_{ijm}}{T_{ijm}-\gamma_{jm}} - \lambda_i w^*_{im} = 0, \text{ for } j = l+1, \ldots, J \quad \text{(A5b)}$$

$$\frac{\alpha_{ijf}}{T_{ijf}-\gamma_{jf}} - \lambda_i w_{if} = 0, \text{ for } j = 1, \ldots, J \quad \text{(A5c)}$$

$$\frac{\alpha_{iq}}{q_i-\gamma_q} - \lambda_i = 0. \quad \text{(A5d)}$$

Note that in equation (A5) the man's market wage, w_{im}, has been replaced by the reservation wage, w^*_{im}. Dividing by the mth first-order condition and taking logs gives

$$\begin{aligned} P(w^*_{ijm} \leqslant w^*_{im}) = {} & P(\varepsilon_{ijm} \leqslant \log(-\gamma_j) - \log(T_{iJm}-\gamma_{Jm}) - Z_i'\beta_{jm}), \\ & \text{for } j = 1, \ldots, l \end{aligned} \quad \text{(A6a)}$$

$$\begin{aligned} \varepsilon_{ijm} = {} & \log(T_{ijm}-\gamma_{jm}) - \log(T_{iJm}-\gamma_{Jm}) - Z_i'\beta_{jm}, \\ & \text{for } j = l+1, \ldots, J-1 \end{aligned} \quad \text{(A6b)}$$

$$\begin{aligned} P(w^*_{im} \geqslant w_{im}) = {} & P(\varepsilon_{iq} \leqslant \log(q_i-\gamma_q) - \log(T_{iJm}-\gamma_{Jm}) \\ & - \log(w_{im}) - Z_i'\beta_q) \end{aligned} \quad \text{(A6c)}$$

$$\begin{aligned} \varepsilon_{iq} - \varepsilon_{iJf} = {} & \log(q_i-\gamma_q) - \log(T_{iJf}-\gamma_{Jf}) \\ & - \log(w_{if}) - Z_i'(\beta_q - \beta_{Jf}) \end{aligned} \quad \text{(A6d)}$$

$$\varepsilon_{ijf} - \varepsilon_{iJf} = \log(T_{ijf} - \gamma_{jf}) - \log(T_{iJf} - \gamma_{Jf}) - Z_i'(\beta_{jf} - \beta_{Jf}), \quad \text{for } j = 1, \ldots, J-1. \tag{A6e}$$

Thus, the contribution to the likelihood of a household in this case is

$$\begin{aligned} L_{i3} = {} & P(w_{i1m}^* \leqslant w_{im}, \ldots, w_{ilm}^* \leqslant w_{im}, w_{im}^* \geqslant w_{im} | T_{il+1m}, \ldots, \\ & T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i, Z_i, w_{if}, w_{im}) \\ & \times f_3(T_{il+1m}, \ldots, T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i | Z_i, w_{if}, w_{im}) \end{aligned} \tag{A7a}$$

$$\begin{aligned} = {} & P(w_{i1m}^* \leqslant w_{im}, \ldots, w_{ilm}^* \leqslant w_{im}, w_{im}^* \geqslant w_{im} | T_{il+1m}, \ldots, \\ & T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i, Z_i, w_{if}, w_{im}) \\ & \times f_{3a}(\varepsilon_{il+1m}, \varepsilon_{iJ-1m}, \varepsilon_{i1f}, \varepsilon_{iJ-1f} | Z_i, w_{if}, w_{im}) \left| \frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i} \right|, \end{aligned} \tag{A7b}$$

where $P(.)$ is an $l+1$ variate normal distribution function; f_3 is the joint density of $\tilde{T}_i = (T_{il+1m}, \ldots, T_{iJ-1m}, T_{i1f}, \ldots, T_{iJ-1f}, q_i)$ conditional on the regressors Z_i and the wages w_{if} and w_{im}; f_{3a} is the multivariate normal density function of $\tilde{\varepsilon}_i = (\varepsilon_{il+1m}, \varepsilon_{iJ-1m}, \varepsilon_{i1f}, \varepsilon_{iJ-1f})$; and $|\partial\tilde{\varepsilon}_i/\partial\tilde{T}_i|$ is the absolute Jacobian from $\tilde{T}_i$ to $\tilde{\varepsilon}_i$. $|\partial\tilde{\varepsilon}_i/\partial\tilde{T}_i|$ has a similar structure to (A4). The likelihood can be formed by combining the probabilities given in (A3) and (A7).

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