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Private Equity Fund Returns and Performance Persistence*

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Abstract. Successful private equity managers have funds that are often oversubscribed and provide persistent abnormal returns. Why do not successful managers increase fund size or fees? We argue that managers want to attract high-quality entrepreneurs, while entrepreneurs want to match with high-ability managers. However, observing fund performance does not allow entrepreneurs to distinguish a manager's ability from the quality of firms in the fund's portfolio. As a consequence, a fund manager may devote unobserved effort to select firms, and keep fund size small to limit the cost of effort, hoping to manipulate entrepreneurs' beliefs about his ability.

JEL Classification: G24, G31

1. Introduction

Anecdotal evidence indicates that follow on funds of successful private equity managers tend to be oversubscribed, suggesting that managers restrict fund size and decline some of the money investors are willing to provide.[1] Given diseconomies of scale in private equity (Lopez-de-Silanes, Phalippou, and Gottschalg, 2009; Metrick and Yasuda, 2010), restricting fund size may enhance returns delivered to investors but may reduce the size of assets under management and total fee income. Recent evidence also shows that, although private equity funds may not deliver abnormal returns on average, successful managers generate persistent abnormal

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[1] Several cases in which VC funds were oversubscribed are noted in the article "Oversubscribed", *European Venture Capital Journal*, November 2006.

returns for their investors (Kaplan and Schoar, 2005; Phalippou and Gottschalg, 2009).[2] This can be contrasted to the case of open-end mutual funds that have few restrictions on investor flows and do not exhibit performance persistence (e.g., Jensen, 1968; Malkiel, 1995). Performance persistence is also observed in hedge funds (Jagannathan, Malakhov, and Novikov, 2010), and Glode and Green (2011) provide an explanation based on managers delivering excess returns to investors to induce them to not divulge information about the fund's investment strategy. However, similar concern for confidentiality of the investment strategy may be less important in private equity, especially given that the assets in private equity (PE) firms' portfolios are often not publicly traded.

We are left with an obvious question with regard to private equity funds: Why do not successful private equity managers capture higher rents by increasing the size of their follow-on funds rather than leaving abnormal returns for their investors? To address this question, we focus on a fundamental difference between private equity and mutual funds. Unlike mutual funds that invest in public securities, investments by private equity funds are subject to a two-sided matching problem (Sorensen, 2007).[3] Private equity funds seek to invest in high-quality entrepreneurial firms while, on the other side, entrepreneurs try to pair with talented fund managers that are more likely to add value.[4]

Potential entrepreneurs can learn about a manager's ability from the past performance of his funds. A complication, however, is that in addition to his ability to add value a manager's performance is also affected by the innate quality of the firms in his portfolio. The contribution of these two factors to fund performance is hard to disentangle, especially since the perceived ability of a fund manager and the quality of firms in his portfolio are not independent of each other. Indeed, Sorensen (2007) provides evidence that both effects are important in explaining venture capital (VC) fund success. Our premise is that managers can improve the quality of their matches by

[2] It is not clear whether performance persistence could be easily exploited ex-ante (Lerner, Schoar, and Wong, 2007).

[3] Several papers highlight the importance of matching in the context of financial intermediation (e.g., Chemmanur and Fulghieri, 1994, and Fernando, Gatchev, and Spindt, 2005).

[4] Fund managers may add value through providing strategic advice, helping to professionalize firm management, by attracting better resources and increasing the probability of an IPO (Gorman and Sahlman,1989; Megginson and Weiss,1991; Hellmann and Puri, 2000, 2002; Baum and Silverman, 2004; Ozmel, Robinson, and Stuart, 2013). In line with this, Hsu (2004) finds that firms are more likely to accept an offer—even if the terms are less attractive—when a VC is more reputable and, presumably, has greater ability to add value.

expending unobserved costly effort. Since this effort is not observed, the manager is asymmetrically informed about the source of fund returns and, in equilibrium, will try to manipulate the beliefs of entrepreneurs.

The model we present incorporates important features of the actual fund raising process. It is well known that the process of raising a fund usually takes place over several stages. Typically, fund managers (general partners) set fund fees and a target fund size before going on a road show to attract capital from institutional investors, which may take 10–18 months (Burton and Scherschmidt, 2004; Ramsinghani, 2011). At the end of the road show, depending on investor enthusiasm, the fund may be over- or under-subscribed relative to the initial target size, or may never be formed. If the fund is oversubscribed, which reveals a positive collective assessment of fund's investment opportunities and future potential returns, the general partner is often allowed to increase the fund's size.[5]

We incorporate these institutional details into an infinite period model in which managers can introduce a new fund each period. Each period encompasses three stages of capital raising and investment. Specifically, in the first stage of each period the manager sets up a fund by determining what fees to charge. During the fund raising process, which represents the second stage, all parties learn more about investment opportunities available to the manager, and competitive investors use this information to decide how much money to actually invest in the fund. The manager then uses the information about investment opportunities and investor demand to determine the fund's size and the optimal amount of effort to exert in selecting target firms. In the third stage, the fund's return is revealed.

As a base case, we consider the situation in which a manager's effort is publicly observed. In this case, the manager cannot manipulate investor beliefs about his ability and, as we show, investors do not earn excess returns since the fund manager increases the fund's size to extract all the surplus from investors. In effect, this is similar to the result in Berk and Green (2004) that competitive investors drive down expected excess return to zero, with mutual fund managers accepting all the funds provided by investors.

We then move to a more realistic setting in which the fund manager's effort is not observed by outsiders. In this case, the manager finds it optimal to exert additional effort in searching and matching with higher quality firms in the hope of manipulating the beliefs of entrepreneurs about his ability. The marginal cost of effort increases with the size of the

[5] See Lerner, Schoar, and Wong (2007) for a detailed description of this process and contracts that govern it.

fund, so that beyond a certain size it becomes too costly to try to manipulate beliefs by providing higher returns. We show that this limits the extent of investor funds that the manager will accept, even if the investors expect to receive excess returns.

Since the manager may not accept all the funds that investors are willing to provide, fund returns for investors will be positive in expectation. This is despite the fact that the manager chooses the fund's fees and size optimally for each consecutive fund he raises. The manager faces a similar trade-off each time he sets up a fund, giving rise to positive expected performance persistence over time (i.e., for funds formed by the same manager). Managers with higher ability can add more value by matching with better entrepreneurs, which gives them a greater incentive to manipulate entrepreneurs' beliefs. Therefore, these managers limit fund size further, leading to return predictability in the cross section of managers. Hence, managers that have done relatively better in the past are more likely to do relatively better in the future, which is consistent with the empirical evidence (Kaplan and Schoar, 2005; Phalippou and Gottschalg, 2009).[6]

Our framework is based on the seminal work by Holmstrom (1999) on "signal jamming". Stein (1989) is one of the first to apply this setting to financial markets. In these models, an agent who does not know his own ability uses an unobserved action to try to affect the principal's perception of his ability by manipulating a signal.[7] In our context, the principal is the set of future entrepreneurs and the signal is past fund returns. In equilibrium, however, entrepreneurs are not fooled since they recognize the manager's incentives and can rationally anticipate his actions. Nevertheless, the manager is forced to try to manipulate entrepreneurs' beliefs since he would otherwise face a higher risk of being assessed as having low ability.

Beyond explaining performance persistence, we can explain related empirical evidence. Managers that experience a larger (positive) shock to their investment opportunity set will raise larger funds as well as provide higher returns to investors despite decreasing returns to scale. This is consistent with the positive correlation between fund size and returns in the cross section found in Kaplan and Schoar (2005). However, as uncertainty about a manager's ability decreases, the manager's incentive to manipulate

[6] This reflects an important difference from mutual funds. For successful funds, Carhart (1997) shows that "common factors in stock returns and investment expenses almost completely explain persistence in equity mutual funds' mean and risk-adjusted returns" (p. 57). The worst performing mutual funds do seem to exhibit some performance persistence (see Carhart, 1997), possibly resulting from inattention by investors in these funds.

[7] The manager cannot signal his ability since he does not know it, which distinguishes signal-jamming models from signaling models.

the beliefs of entrepreneurs decreases. As a result, we predict that fund size and fees increase and performance decreases over time for a given manager. To the best of our knowledge, this implication is unique to our framework and is consistent with empirical findings that fees increase (Robinson and Sensoy, 2013) and fund returns decrease in consecutive funds (Kaplan and Schoar, 2005) of the same manager. In our model, performance persistence is driven by private equity fund managers' desire to attract good investment opportunities. This may explain why mutual funds that also cater to institutional investors do not exhibit performance persistence (Busse, Goyal, and Wahal, 2010) given that they invest in publicly traded securities. Likewise, variation in performance persistence across different types of private equity funds can be explained by variation in managers' abilities to manipulate the beliefs of entrepreneurs. For instance, there is little evidence of persistence for those funds that focus on buyout rather than venture financing (Kaplan and Schoar, 2005; Harris, Jenkinson, and Kaplan, 2014). Matching is likely more important in VC investing, where entrepreneurs worry about a fund manager's ability to add value, compared to buyout investing, where targets are more concerned with just the buyout price.

In addition, our model provides several new, testable predictions. We expect higher performance persistence when managers' incentives to manipulate the beliefs of entrepreneurs are larger. This happens when the impact of a manager's perceived ability on the matching process is higher, when the impact of effort on the quality of the firms in the fund's portfolio is greater, and when there is greater uncertainty about a manager's ability. We speculate that these predictions could be tested using cross-sectional differences among private equity funds in terms of their focus on early versus later stage investment, on lead versus nonlead position and on investing in firms versus other funds. Greater performance persistence is expected for funds that take lead positions in early stage investments in firms.

We show that our model can readily be extended to account for the zero or even negative aggregate returns in private equity documented by, among others, Phalippou and Gottschalg (2009).[8] Such negative returns are puzzling given the increasing amount of investments in private equity and performance persistence in returns. We argue that, if investors provide funding to inexperienced and poorly performing funds to tacitly obtain

[8] Whether aggregate private equity excess returns are negative or positive is still under debate. See also Quigley and Woodward (2003), Jones and Rhodes-Kropf (2003) and Cochrane (2005), Hwang, Quigley, and Woodward (2005), Kaplan and Schoar (2005), Korteweg and Sorensen, (2010), Faccio *et al.* (2011), Robinson and Sensoy (2011), Harris, Jenkinson, and Kaplan (2014) and Phalippou (2014) for this debate.

the right to invest in future funds of the managers who are successful, this will result in negative aggregate returns during times when there are many new managers entering the industry.

The assumptions regarding the fund raising process, which we borrow from actual practice, introduce a rigidity in that fund fees, while set optimally given ex ante information, are not adjusted once uncertainty resolves. In other words, the manager cannot raise fees ex post to capture a higher surplus. While this rigidity allows the manager's "signal jamming" to affect excess returns and deliver persistence in performance, it does not actually prevent the manager from extracting the entire surplus from investors by increasing the fund's size, as shown in Section 4 for the case where all information is symmetric.

One can apply our intuition to different settings where signal-jamming toward investors instead of entrepreneurs might be important. For instance, hedge fund managers may wish to manipulate portfolio risk to provide higher returns and attract investors. Similarly, investment banks may try to affect their clients' perceptions of their expertise. For instance, if clients pay attention to past deal volume, banks could inflate volume by providing benefits to the issuer that are not entirely transparent to outsiders, such as guaranteeing analyst coverage, market-making after the IPO, or cutting fees.

2. Related Literature

Our model is largely based on Berk and Green (2004), who explain the lack of performance persistence in the mutual fund industry by arguing that managers with higher ability attract investor flows which, given diseconomies of scale, erodes their ability to provide excess returns. Berk and Green (2004) highlight the conflict of interest between fund managers and investors regarding fund size when managerial fees depend on the funds under management. Our model differs on two key issues. First, we assume that there is positive assortative matching, that is, there tends to be a pairing between managers with higher (perceived) ability and better entrepreneurs, as identified in the empirical literature on private equity investments (Hsu, 2004; Sorensen, 2007). The second ingredient is more subtle: the manager can take an unobserved action that affects fund returns, and specifically can exert effort to locate higher quality firms. Indeed, when effort is observed and the manager cannot manipulate the beliefs of investors, we obtain similar results to those in Berk and Green (2004).

Fund managers may capture the surplus they generate through increasing fees or increasing the fund's size. Indeed, an important piece of the performance persistence puzzle for VC funds is that follow-on funds of successful fund managers are often oversubscribed, so that not only do managers not increase fees to extract greater surplus from investors, they also limit fund size for any given level of fees. This latter mechanism has, to our knowledge, been little explored in the literature, that has focused primarily on bargaining or asymmetric information problems between fund managers and investors that induce managers to share surplus with limited partners in their funds, as we discuss below.

Glode and Green (2011) offer an explanation for performance persistence in hedge funds. In their model, hedge fund managers deliver excess returns to investors as a way of providing them with incentives to not share the fund's investment strategy with others. In other words, information obtained by investors endows them with bargaining power over managers. As Glode and Green (2011) emphasize, their approach is based on a concern for confidentiality and applies more to settings where funds' proprietary trading strategy or sector focus may be more easily replicated. Confidentiality is less of a concern in private equity, however, where investments are observed but cannot be easily replicated by other managers. This latter setting is captured more naturally in our model, where the focus is on managerial ability as a key determinant of performance. This is likely more suitable for the private equity industry where value creation is perhaps more related to improvements in firm management and strategy rather than, say, identifying mispriced publicly traded assets.

A contemporaneous paper by Hochberg, Ljungqvist, and Vissing-Jorgensen (2014) argues that institutional investors obtain "soft information" concerning a private equity fund manager's skill after they invest and can hold up the manager (as in models of informed bank lending, such as Rajan, 1992) by refusing to invest in consecutive funds. The bargaining power investors gain is manifest in performance persistence between consecutive funds. A proviso is that incumbent investors do not compete away follow-on funds' excess return due to risk aversion, for instance. In contrast, investors are risk-neutral and competitive in our model, as in Berk and Green (2004). Our focus is on a manager's decision concerning fund size for any given level of fees, while still allowing fees to change (and be set optimally) over time as information concerning the fund manager's ability becomes available. Another important difference is that we allow fees to depend explicitly on fund size or performance, as is observed in practice, which introduces a possible conflict of interest between the fund manager and the investors regarding fund size. This approach allows us to not only explain performance persistence but also related empirical evidence that is hard to reconcile with explanations based solely on changes in the bargaining power of investors. For instance, we explain why managers may limit fund size when they are oversubscribed, why total fees go up for consecutive funds (Robinson and Sensoy, 2013), why returns to investors on consecutive funds decrease over time (Kaplan and Schoar, 2005) instead of increasing as would obtain from increased bargaining power of investors. Since our model is dynamic and set in an infinite-time horizon, it can also explain why performance may persist even for nonoverlapping funds (Kaplan and Schoar, 2005), that is, after investors observe hard information about the prior fund's returns.

A key distinguishing element of our approach is that we focus on the investment side of private equity, in contrast to work that focuses on the investor side (Hochberg, Ljungqvist, and Vissing-Jorgensen, 2014; Glode and Green, 2011). The need for this complementary approach is supported by the emerging evidence indicating that differences in performance persistence in private equity and mutual funds are unlikely to be entirely driven by differences in bargaining power of individual investors and institutional investors. For example, mutual funds that cater to institutional investors do not exhibit performance persistence (see Busse, Goyal, and Wahal, 2010). Likewise, performance persistence varies across different types of private equity funds, with little evidence of persistence for those funds that focus on buyout rather than venture financing (Kaplan and Schoar, 2005; Harris *et al.*, 2013). In our model, differences between private equity and mutual funds are driven by differences in investments, in particular private equity managers' desire to attract good investment opportunities. VC funds have an incentive to manipulate the beliefs of prospective entrepreneurs by expending costly effort to select higher quality firms and improve performance. These incentives would not exist when investing primarily in public securities and may explain the lack of performance persistence in mutual funds that cater to institutional investors. On the other hand, these incentives would be stronger for VC funds, for whom matching with their portfolio firms is important, than for buyout funds, whose targets are more concerned with just the final buyout price.

Our results also contribute to the literature that analyzes the optimal size of VC firms. For instance, Inderst, Mueller, and Muennich (2007) argue that limiting size benefits the VC by weakening the bargaining position of portfolio firms. Fulghieri and Sevilir (2009) show that a VC may limit a fund's size when it is important to provide entrepreneurial incentives to firms. A small portfolio increases the value-added to each firm by the VC and encourages entrepreneurs to exert higher effort. In contrast, in our case a fund manager limits the fund's size to reduce the cost of manipulating the beliefs of future entrepreneurs.

3. Model

Assume an infinite horizon, with each period denoted by t. Within each period there are three stages, representing the life cycle of a private equity fund. At the beginning of each period t (stage 1), a VC fund manager prepares a private placement memo for investors, which specifies the targeted fund size and the fund's fees f_t.

The fund raising process can be time consuming and the evidence indicates that the fund raising process may take anywhere from 10 to 18 months (Burton and Scherschmidt, 2004; Ramsinghani, 2011). To model the passage of time, we assume that in stage 2, both the manager and the fund investors learn more about the average quality of investments available for the manager and the perception of the investment community about the fund's potential returns. Consequently, investors decide how much they are willing to invest and the manager chooses his level of effort and determines the actual fund size.[9] Fund returns are realized at the end of period t (stage 3), and are observed by all parties: entrepreneurs, investors, and the fund manager. The process repeats itself every period.[10] Figure 1 summarizes the timing of events.

We denote a fund manager's ability by X, which is distributed according to $N(\overline{X}, \sigma_X^2)$. The fund's investment, coupled with the manager's ability, results in a value added of $X + \varepsilon_t^X$ percent in period t, where ε_t^X is a manager-specific random shock to the value added he generates. The shock ε_t^X is *i.i.d.* over time and is distributed as $N(0, \sigma_\varepsilon^2)$. The actual realizations of X and ε_t^X are unknown to the fund manager, fund investors, and entrepreneurs. The random shock ε_t^X introduces noise so that investors cannot perfectly back out managerial ability X from realized fund returns.

Assuming that the manager learns about his abilities at the same time as everyone else (see, e.g., Holmstrom, 1999 or Stein, 1989) allows us to abstract from issues of signaling by managers. Hence, there is gradual learning about the manager's abilities based on the fund's performance over time.[11] All

[9] We assume that any new information that arrives during the process of raising capital cannot be verified by a court. Hence, the terms on which the fund raises capital are incomplete in the sense that they cannot be made contingent on new information.

[10] The assumption that the number of periods is infinite simplifies the exposition since it makes the value function stationary. Assuming either a finite number of periods or that there is a probability each period of the manager exiting permanently does not qualitatively affect our results. In Section 6.1, we use a three-period model to show that our model is consistent with empirical evidence on the average returns from PE investments.

[11] Market participants might also obtain information about managerial ability in different ways—for instance, by observing the value of individual portfolio firms that have gone through an IPO. All of our intuition and proofs go through if investors use returns to

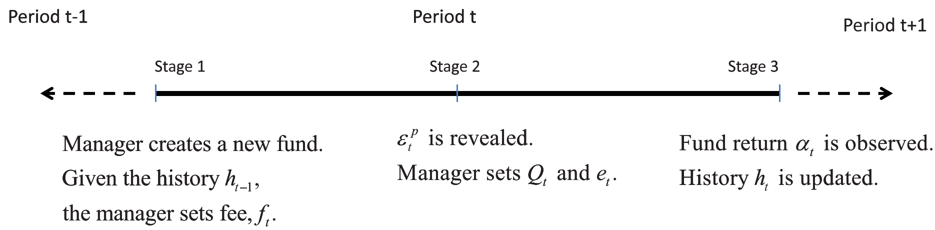

Figure 1. Timeline with sequence of events (three stages) in each period of infinite horizon game.

market participants update their expectations of the manager's ability, $E_t[X|h_{t-1}]$, using the history h_{t-1} that is available at the beginning of time t. The history h_{t-1} consists of all past fund returns, fund fees, and fund sizes. To economize on notation, we will use $E_t[X] \equiv E[X|h_{t-1}]$.

We assume that the private equity fund is characterized by decreasing returns to scale as has been documented by, for instance, Lopez-de-Silanes, Phalippou, and Gottschalg (2009). The decreasing returns to scale assumption captures the idea that a manager's ability to add value is not easily scalable, perhaps due to constraints on his human capital and time. We denote this cost by $S(Q_t)$, where Q_t is the amount of invested funds. This cost is independent of ability and is increasing and convex in Q_t (as in Berk and Green, 2004). After incorporating the per unit cost of generating value $S(Q_t)$, the gross percentage value added for a dollar of investment is proportional to $W_t = X + \varepsilon_t^X - S(Q_t)$.

On the investment side, entrepreneurs[12] of varying quality need to raise a fixed amount of financing, normalized to \$1, in return for giving a fraction of the company to the investing VC fund. The quantity of shares and the price at which a firm's shares are sold affect how any value that is created is shared between the fund and the entrepreneur. We abstract from the details of the bargaining between the entrepreneurs and the fund manager and assume that both emerge with an equity stake in the firms. Hence, both parties receive a strictly positive share of the value created. A higher quality firm is defined as one in which a fund's investment, coupled with the fund manager's ability, results in higher value creation.

The expected (average) quality of firms seeking investment, $\overline{P}$, is common knowledge at the beginning of the period. However, by the end of the fund

portfolio firms, rather than the returns to fund investors, to draw inferences about a manager's ability to add value.

[12] We will use the terms "firm" and "entrepreneur" interchangeably to refer to the party receiving an investment from a VC fund.

raising process all parties learn more about the average quality of investments available. We model this by assuming that an innovation "shock" (ε_t^P) to the quality of firms in which the fund can potentially invest becomes known in stage 2 of each period t. The innovation ε_t^P is specific to a manager, *i.i.d.* over time, and is distributed as $N(0, \sigma_P^2)$. The innovation is intended to capture the notion, discussed above, that over the several months that it takes to raise financing, new information regarding the fund manager's investment opportunity set can arrive or limited partners can learn from observing each others' interest in the fund. For example, we know that if there is insufficient demand for the fund, that is, if the manager cannot raise the minimum target fund size, investors that have signed up earlier may ask for their money back (Lerner, Hardymon, and Leamon, 2007). This uncertainty in the fund raising process may result in larger or smaller funds compared to the initial target fund size. A consequence of the uncertainty in the fund raising process reflected in ε_t^P is that the manager cannot capture all the surplus purely through the setting of fees at the beginning of the fund raising process. The manager can, however, adjust the size of the fund in response to this shock and, as we show below, is able to extract all expected surplus from investors through this mechanism. That he may not find it optimal to do so is the main implication of the article.

The final (realized) quality of firms in the fund's portfolio depends on various factors: the average quality of firms that are available to the fund for investment, the shock to the average quality, the perceived ability of the manager, and the manager's effort to select firms. Since managerial ability is associated with a higher value creation and entrepreneurs are presumed to capture some of this value, managers with higher perceived ability are more likely to match with better entrepreneurs. For our purposes, it is sufficient to take a reduced form approach to the determination of the fund's portfolio quality and we define $P_t = z(\overline{P}, e_t, E_t[X]) + \varepsilon_t^P$ as the average quality of firms in the manager's portfolio, where e_t is the managerial effort to select firms. In particular, the quality of firms in the portfolio is an increasing and concave function of effort, and is an increasing function of the expected ability to add value by the fund manager. This implies $\frac{\partial P_t}{\partial e_t} > 0$, $\frac{\partial^2 P_t}{\partial^2 e_t} < 0$, and $\frac{\partial P_t}{\partial E_t[X]} > 0$. The cost of effort C is increasing and convex in effort and is increasing in the size of the manager's portfolio: $\frac{\partial C}{\partial e_t} > 0$, $\frac{\partial^2 C}{\partial^2 e_t} > 0$, $\frac{\partial C}{\partial Q_t} > 0$, and $\frac{\partial C^2}{\partial Q_t \partial e_t} > 0$ for $e_t > 0$, and zero if $e_t = 0$. A cost of effort that increases with respect to fund size is intended to capture the notion that there may be significant constraints on certain resources, such as human capital, that fund managers can allocate to screening firms.

The gross percent return to the fund at the end of the period is equal to $P_t W_t$. The net abnormal return to fund investors after fees is $\alpha_t = P_t W_t - f_t$. Investors are competitive and risk-neutral and, hence, are willing to provide capital as long as their expected net return, $E_t[\alpha_t]$, is nonnegative. We define returns in excess of what investors require to participate as abnormal returns.

The manager chooses his actions to maximize his expected payoff subject to the participation constraint of the investors. The total expected future payoff for the fund manager as of time t can be expressed as:

$$V_t = E_t\left[\sum_{i=t}^{\infty} \delta^i (Q_i f_i - C_i)\right], \tag{1}$$

where $\delta < 1$ is the discount factor, which we ignore in the rest of the article for brevity.

Before we begin the analysis, it is useful to summarize the differences and commonalities in the information sets of investors, entrepreneurs, and the manager. All fund characteristics including fund size, fees, and functional forms are common knowledge. The realized fund return is publicly observed at the end of the period, while the random shock to the average quality of firms (ε_t^P) is publicly observed during the fund raising process (stage 2), although it is noncontractible. We use the fund's history at the beginning (i.e., stage 1) of period t to summarize the common knowledge of fund investors and entrepreneurs, which always includes all the realized returns α_i and fund characteristics for $i < t$. On the other hand, no one knows the manager's true ability to add value X or the per period shock ε_t^X. The only difference in information between the fund manager and the outsiders (entrepreneurs and limited partners) is that the manager always knows his effort. As a result, entrepreneurs that observe $P_t W_t$ have to disentangle whether higher returns comes from a higher screening effort ("selection", reflected in higher P_t) or whether the managers have a greater ability to add value ("treatment", reflected in higher W_t).

4. Observed Managerial Effort

As a starting point, we analyze the case in which managerial effort is observed, so there is no information asymmetry between the fund manager, investors, and entrepreneurs. Hence, all market participants can back out the value added W_t in that period. This symmetric information setting provides a baseline case and helps put into sharper focus our subsequent discussion of the case with private information. The symmetric information case is similar to that studied in Berk and Green (2004).

We solve the model by backward induction. Consider first how entrepreneurs update their beliefs regarding managerial talent. By the end of period t (i.e., at stage 3), entrepreneurs have observed the manager's choices of fund size Q_i, fees f_i, managerial effort e_i, and the per-period innovation to average firm quality ε_i^P, for $i \leq t$. They have also observed the total fund returns $P_i W_i$, for $i \leq t$. Entrepreneurs can use the information on managerial effort e_i and the innovation ε_i^P to infer P_i, and then use the fund's total return to get W_i. Since $W_i = X + \varepsilon_i^X - S(Q_i)$, knowledge of $S(Q_i)$ and W_i provides entrepreneurs with a noisy signal of the manager's ability, $X + \varepsilon_i^X$, for each period $i \leq t$. Bayesian updating now gives us the conditional expectation of X at the beginning of period $t+1$ as:

$$E_{t+1}[X] = w_t E_t[X] + (1 - w_t)\left(X + \varepsilon_t^X\right). \tag{2}$$

The weights w_t reflect the importance of the most recent realization of fund returns relative to the past history in updating entrepreneurs' expectations concerning X. The weight assigned to new information, $1 - w_t$, decreases over time through Bayesian updating.[13]

After investor demand has been observed and ε_t^P is revealed, the manager simultaneously decides on his effort level and fund size. In equilibrium, investors make their funds available only if their participation constraint is satisfied, given the level of effort chosen by the manager and the size of the fund being managed. Given that managerial actions have no effect on prospective entrepreneurs' beliefs about the manager's ability in future periods, the manager simply maximizes his current period payoff. The maximization problem becomes

$$\max_{Q_t, e_t} Q_t f_t - C_t, \tag{3}$$

subject to the participation constraint of fund investors,

$$E_t[\alpha_t | \varepsilon_t^P] = E_t[P_t W_t] - f_t \geq 0. \tag{4}$$

Note that the expectation here is taken at stage 2, after observing ε_t^P.

[13] Given that the underlying distributions are normal, the weight placed on the new information in Bayesian updating will be determined by the precision of the new information relative to the precision of the prior. Specifically, if the conditional distribution of X at $t - 1$ is denoted by $N(E[X|h_{t-1}], \sigma^2_{X,t-1})$, we have $1 - w_t = \frac{1/\sigma_\varepsilon^2}{1/\sigma_\varepsilon^2 + 1/\sigma^2_{X,t-1}}$. It is apparent that as the precision of the conditional distribution of X increases through time, the weight placed on the new information will correspondingly decrease.

Proposition 1
When managerial effort is observable, for any given $f_t > 0$ the manager chooses a fund size Q_t and effort e_t such that fund investors' expected abnormal return, $E_t[\alpha_t|\varepsilon_t^P]$, is zero. There is also a value $\underline{\varepsilon}_t^P$ such that no fund is raised for $\varepsilon_t^P < \underline{\varepsilon}_t^P$. In this case, all parties' expected returns are zero.

When the manager's effort level is observed, the manager is unable to affect entrepreneurs' beliefs concerning his talent. Hence, as in Berk and Green (2004), the manager captures the entire surplus by accepting all the funds that investors are willing to provide. In other words, the investors' participation constraint binds in equilibrium and investors' expected (excess) return is zero. To see this, suppose that the participation constraint did not bind. In this case, the manager would always be better off by accepting more money from investors and/or reducing effort level. Decreasing returns to scale ensures that the participation constraint of investors will eventually bind as fund's size increases. This argument applies to any arbitrary time period t and implies that, when effort is observed, there is no performance persistence over time for the funds of the same manager or cross-sectional performance persistence across managers.

We now consider the manager's problem in determining the optimal fee in the first stage. The fee is optimally chosen by the manager to maximize his surplus for the current period:

$$\max_{f_t} E[Q_t f_t - C_t], \tag{5}$$

subject to fund size Q_t and effort e_t being chosen optimally in the second stage (i.e., as the solutions to (3) in Proposition 1). There are several considerations in selecting the optimal fee. For a given fund size, the manager's payoff increases as fees increase, and the equilibrium level of effort increases as well. However, the optimal size for the fund is decreasing in the fees because the investors' participation constraint binds (see Proposition 1). Further, a larger fee increases the probability that, after ε_t^P is revealed, investors' participation constraint will not be satisfied for any combination of fund size and effort, so that the fund cannot be established. As a result, the optimal fee $f_t^* < \infty$ balances larger fees per dollar with a lower fund size, a higher cost of effort and a lower probability of establishing the fund (a proof is provided in the Appendix).

As a final point, note that the manager determines the optimal fee for each consecutive fund by considering all the information available at the time. As a result, the optimal fee changes over time as market participants update their beliefs about the manager's ability.

5. Unobserved Effort

We next analyze the more realistic case in which the fund manager's effort to select firms is not observed by outsiders and discuss empirical implications of our model.

5.1 SIGNAL-JAMMING

The difference between the case studied here and that in Section 4 is that, while the total return $P_t W_t$ is still observed at the end of investment period t, entrepreneurs do not directly observe the effort e_t exerted by the fund manager. This prevents them from being able to perfectly back out the value $X + \varepsilon_t^X$. In order to make an assessment of the manager's ability, entrepreneurs must conjecture the level of managerial effort, and we denote such a conjecture as e_t^c. From Bayesian updating, the conditional expectation of X at the beginning of period $t+1$, after observing the total return $P_t W_t$ realized at the end of period t, is

$$E_{t+1}[X] = w_t E_t[X] + (1 - w_t)\left(\frac{P_t W_t}{P_t(h_t, e_t^c)} + S(Q_t)\right), \tag{6}$$

where we use $P_t(h_t, e_t^c)$ to denote entrepreneurs' beliefs about the average quality of firms in the fund's portfolio, given the conjectured effort level e_t^c. Equation (6) can be written as:

$$E_{t+1}[X] = w_t E_t[X] + (1 - w_t)\left(\frac{P_t(h_t, e_t)(X + \varepsilon_t^X - S(Q_t))}{P_t(h_t, e_t^c)} + S(Q_t)\right). \tag{7}$$

It is clear from (7) that $\frac{\partial}{\partial e_t} E_{t+1}[X] > 0$ because $E_t[X + \varepsilon_t^X - S(Q_t)]$ must be positive when the manager charges a positive fixed fee. Otherwise, investors would earn negative returns in expectation and would choose not to participate. Hence, given the conjectured level of effort, entrepreneurs' beliefs $E_{t+1}[X]$ are increasing in the manager's actual effort in period t. This adds an important dimension to the manager's decision concerning the fund's fees, size, and his effort level since he now has to consider the effect of his current decisions on future payoffs through their influence on entrepreneurs' beliefs about his ability. In other words, the manager's optimization problem becomes dynamic.

After the fund raising stage is over (and ε_t^P is realized), the fund manager's maximization problem in stage 2 of period t can be expressed in the form of a Bellman equation:

$$V_t(E_t[X]) = \max_{Q_t, e_t} E_t[Q_t f_t - C_t + V_{t+1}(E_{t+1}[X])], \tag{8}$$

with the requirement that the participation constraint of fund investors must be satisfied,

$$E_t[\alpha_t|\varepsilon_t^P] = E[P_t W_t] - f_t \geq 0. \tag{9}$$

Note that (9) can never be satisfied for a manager with $E_t[X] \leq 0$, so that no fund would be formed. In what follows, we assume throughout that $E_t[X] > 0$. In other words, we focus on the cases where a given fund manager's perceived ability is sufficiently high to have some chance of establishing a fund.

Lemma 1

The fund manager optimally limits the size of the fund: there is a value $Q_t^{\max}$ such that $Q_t^* \leq Q_t^{\max}$, and optimal effort is $e_t^* \geq e_t^{\min} > 0$ for all ε_t^P.

The lemma establishes that, rather than continuing to increase the fund's size in response to positive innovations to investment opportunities, the fund manager finds it optimal to limit the size of the fund so as not to exceed some threshold size, regardless of how much investors are willing to invest. The intuition is simple. First, the manager always finds it optimal to exert some effort $e_t^* > 0$ to increase the quality of firms in the portfolio, hoping that by doing so he can influence entrepreneurs' beliefs about his ability. This incentive arises precisely because, given the manager's effort is not observed, higher effort leads to higher fund returns and therefore feed back into entrepreneurs' expectations of the manager's ability. The larger the size of the fund, the greater is the effort cost that the manager will have to bear. However, anticipating the impact of fund size on effort costs, the manager will seek to limit the size of the fund. This leads to an (optimal) upper limit on fund size, $Q_t^{\max}$, which is determined by equalizing marginal revenue from fees, f_t, with the marginal cost of increasing fund size, $\frac{\partial C}{\partial Q_t}$.

Since the upper limit $Q_t^{\max}$ is independent of investors' participation constraint, this introduces the possibility that the manager may choose a fund size that is smaller than the amount investors are willing to invest. Investors' participation constraint will not bind at $Q_t^{\max}$ precisely when the average quality of firms available for investment is high (i.e., when ε_t^P is high). In this case the manager chooses the fund's size to be $Q_t^{\max}$, and chooses a level of effort $e_t^{\min} > 0$. Since the participation constraint does not bind at these choices of size and effort, by definition fund investors expect to receive excess returns.

The finding that a fund manager optimally decides to limit the size of his fund, no matter how good current conditions look, is consistent with the anecdotal evidence that some funds do not increase their size despite being oversubscribed. Moreover, it is useful to note that this is not a constraint imposed on the solution to the model, but rather an optimal choice by the fund manager when effort is unobservable. This is important as it implies that, while the fund manager always has the ability to extract all surplus by increasing the size of the fund, as in Section 4, he finds it optimal not to do so, no matter what initial fee f_t has been chosen. We discuss further this issue below.

Proposition 2
When effort is not observable, for any given fee $f_t < \infty$*, there is a unique equilibrium where, for* $\varepsilon_t^P > \varepsilon_t^{P,low} \equiv \frac{f_t}{E[W(Q_t^{\max})]} - z(e_t^{\min})$*, fund investors' expected abnormal return* $E_t[\alpha_t|\varepsilon_t^P]$ *is strictly positive, and is zero otherwise. There is also a value* $\underline{\varepsilon}_t^P$ *such that no fund is raised for* $\varepsilon_t^P < \underline{\varepsilon}_t^P$.

The proposition characterizes exactly when the upper limit on fund size becomes relevant, which corresponds to when investors' participation constraint does not bind, that is, when investors' expected return is positive. This is accomplished by finding the threshold $\varepsilon_t^{P,low}$ such that investors' expected returns will be zero when the manager chooses $(Q_t^{\max}, e_t^{\min})$. If the average quality of available investments (ε_t^P) is larger than this threshold, the expected returns of fund investors are positive because the manager will prefer to keep the fund's size no larger than $Q_t^{\max}$ to capture higher surplus from fund investors.

If the average quality of investment opportunities is in an intermediate range, so that the funds investors are willing to provide are less than $Q_t^{\max}$, the manager finds it optimal to choose a fund size and effort that just satisfies investors' participation constraint, thus extracting all the surplus in those states. If the investment opportunities are sufficiently poor, specifically when $\varepsilon_t^P < \underline{\varepsilon}_t^P$, there is no fund size for which investors expect to receive a nonnegative return. In these states a fund cannot be established.

Proposition 2 shows that, for any fee $f_t < \infty$, the manager selects there is a threshold $\varepsilon_t^P > \varepsilon_t^{P,\text{low}}$ such that investors' expected return is positive. We close the model by showing that the optimal fee is indeed finite, so that the results from Proposition 2 hold in equilibrium. We next solve the manager's optimization problem when the fee is set. As above, the problem is inherently dynamic, although the manager's choice of the optimal fee does not directly affect entrepreneurs' beliefs about his ability given that fees are observed. The fund manager's objective function at the beginning of period t is:

$$V_t(E_t[X]) = \max_{f_t} \; E_t\left[\int_{\underline{\varepsilon}_t^P} (Q_t f_t - C_t) dN(\varepsilon_t^P) + V_{t+1}(E_{t+1}[X])\right], \qquad (10)$$

where $N(.)$ is the cdf of ε_t^P, which is normally distributed, and Q_t and e_t are chosen optimally, as solutions to (8). We define the solution to (10) under asymmetric information as f_t^A.

Proposition 3

When effort is not observable, the optimal fee $f_t^A < \infty$ balances a larger fee against a lower fund size and a lower probability of establishing the fund. At the optimal fee f_t^A, fund investors' unconditional expected abnormal return, $E_t[\alpha_t]$, is strictly positive. Since market participants learn slowly about managerial talent, there is performance persistence across time for funds established by the same manager. Moreover, $E_t[\alpha_t]$ is increasing in $E_t[X]$, which results in return predictability in the cross section of managers.

The proposition establishes that the optimal fee is bounded, so that the results from Lemma 1 and Proposition 2 are well defined. As the fund manager raises the fee, he obtains greater compensation for every dollar under management. However, raising the fee also reduces the probability that a fund will be formed. The optimal fee f_t^A is set considering these trade-offs, that is, the goal is not to capture all the surplus in all states of the world but to maximize the expected surplus that is captured. This intuition is consistent with the observation that private equity funds fees are affected by various considerations, including agency issues (Robinson and Sensoy, 2013). From Proposition 2 we know that, for any given fees, when ε_t^P is sufficiently high investors' expected return will be positive, and will be zero otherwise. Since this is true for any fees, it is also true at the optimum, implying that fund investors' unconditional expected return at the beginning of the period are positive in equilibrium: $E_t[\alpha_t] > 0$.

Moreover, since the value added by matching with better entrepreneurs is greater for managers with higher ability, a manager's incentive to manipulate the beliefs of entrepreneurs is also increasing in $E_t[X]$. Therefore, managers with higher perceived ability exert higher effort and limit the size of their funds such that, in equilibrium, they provide higher returns to investors. Given that the manager's ability is persistent and he faces a similar problem and incentives anytime he establishes a new fund, this results in persistence in performance as well. Performance persistence here implies that managers that have done relatively well in their previous funds continue to do relatively well in their future funds, which is consistent with the empirical evidence in Kaplan and Schoar (2005).

Finally, note that entrepreneurs are fully rational in our model and recognize the manager's incentive to manipulate their beliefs. As a result, in updating their beliefs about the manager's ability, entrepreneurs correctly anticipate the manager's actions. Therefore, entrepreneurs are not fooled in equilibrium and the conjectured effort is equal to the equilibrium level of managerial effort. Nevertheless, as is typically the case in signal-jamming models (e.g., Stein, 1989; Holmstrom, 1999), the manager cannot avoid the attempt to manipulate entrepreneurs' beliefs by exerting greater effort. This is because he faces entrepreneurs that fully expect him to try to manipulate beliefs, and take that into account in assessing his ability. The manager, therefore, optimally chooses to put in a "manipulative" level of effort because he would otherwise suffer a lower assessment of his ability. As much as he might want to, the manager cannot credibly disclose his effort level given that he has a tendency to report lower effort to exaggerate his ability.

We have taken the fund raising process in private equity as given and explained why fund managers may have incentives to manipulate the beliefs of entrepreneurs. However, given that managers seem to leave some money on the table it is fair to ask what rigidities in this setup provide a channel for the manager's "signal jamming" to operate and result in performance persistence. The main rigidity is that, while fund fees are set optimally at the beginning of every period given the existing information, they cannot be adjusted in the interim, once uncertainty resolves. In other words, the uncertainty in the fund raising process introduced by ε_t^P ensures that the manager cannot capture all the surplus purely through the setting of fees at the beginning of the fund raising process. If this shock is not present, the signal-jamming effect could still arise but it would not translate into higher returns for investors. As discussed above, however, the manager can adjust the size of the fund and is in fact able to extract all expected surplus from investors through this mechanism, but finds it optimal not to do so. This is evident from the analysis in Section 4, where the same assumption about the timing of the setting of fees leads to zero expected returns for investors when managerial effort is observed. In other words, while the assumed rigidity of fees provides a channel for signal-jamming to affect fund returns, it does not actually constrain the manager's ability to extract the entire surplus from investors. While focusing on the timing of fee as rigid is natural given how VC funds tend to be established, we conjecture that other rigidities could deliver similar results.

5.2. COMPARATIVE STATICS AND EMPIRICAL PREDICTIONS

In this section, we provide additional empirical predictions about how fund characteristics and performance persistence varies over time and with respect to various model parameters. Our main prediction is that as a manager's incentive to manipulate the beliefs of entrepreneurs increases, we should observe a higher probability of positive abnormal returns and greater performance persistence. Most of these predictions are obtained directly from the conditionality of our result in Proposition 2 on the average quality of firms available for investment.[14]

Corollary 1
A manager's incentive to manipulate entrepreneurs' beliefs decreases as uncertainty about the manager's ability decreases, that is, as $1 - w_t$ *decreases. As a result fund fees increase,* $Q_t^{\max}$ *increases, and expected returns for consecutive funds of the same manager decrease over time.*

The degree of uncertainty about a fund manager's ability affects his incentives (and opportunity) to manipulate beliefs. As a fund manager's tenure increases, uncertainty about the manager's ability declines over time, and our model predicts that fund returns should go down as fund size and fees increase. These predictions are consistent with the empirical evidence. Robinson and Sensoy (2013) show that as a venture capitalist establishes more funds he charges higher total fees. Kaplan and Schoar (2005) find that, in the time series when they control for general partner (manager) fixed effects, fund sequence and size are negatively correlated with returns to fund investors. To the best of our knowledge, existing theories offer no explanation for this finding. For example, theories that assume increasing bargaining power of investors imply that the returns to investors of subsequent funds should be higher than those of previous funds, so that the time series fund sequence would be positively correlated with fund returns. Our model, which captures the dynamics of fees over a long (i.e., infinite) horizon, also easily explains the evidence on performance persistence for nonoverlapping funds (Kaplan and Schoar, 2005) since the revelation of the fund's final return does not reveal the amount of effort that was spent by the fund manager in delivering those returns.

We next summarize cross-sectional predictions of our model.

Corollary 2

(1) *If a manager's perceived ability has a greater impact on matching, that is, if* $\frac{\partial P_t}{\partial E_t[X]}$ *is larger, abnormal returns to investors will be higher.*
(2) *If effort has a greater marginal impact on the quality of firms in the fund's portfolio, that is, if* $\frac{\partial P_t}{\partial e_t}$ *is larger, or if the marginal cost of effort,*

[14] As we have argued, after the realization of ε_t^P, the expected return will be positive when $\varepsilon_t^P > \varepsilon_t^{P,low} = \frac{f_t}{E[W(Q_t^{\max})]} - z(e_t^{\min})$, and zero otherwise. Intuitively, as this cutoff value $\varepsilon_t^{P,low}$ decreases, the probability that a manager leaves money to investors increases.

$\frac{\partial C_t}{\partial e_t}$, *is lower for every* Q_t, *then the manager will exert higher effort and deliver higher abnormal returns to investors.*

(3) *If the cost of effort is less scalable, that is, if* $\frac{\partial C_t}{\partial Q_t}$ *is larger for every* e_t, *then the manager will choose a smaller fund size and deliver higher abnormal returns to investors.*

(4) *Managers with a higher realization of* ε_t^P *raise larger funds and provide higher abnormal returns to investors.*

These predictions are consistent with empirical evidence. For example, mutual funds catering to institutional investors do not exhibit performance persistence (Busse, Goyal, and Wahal, 2010), which is difficult to reconcile with explanations based on bargaining power of institutional investors over managers. In our framework, performance persistence is driven by private equity managers' desire to attract good investment opportunities (Corollary2, part 1), so we do not expect performance persistence in funds that invest in publicly traded securities. There is also variation in performance persistence observed across private equity funds. Buyout funds exhibit lower performance persistence compared to VC funds (Kaplan and Schoar, 2005, table VII). A manager's perceived ability is likely more important in matching (Corollary 2, part 1) for VC funds than for buyout funds, whose targets are more concerned with just the final buyout price.[15] Empirically, Metrick and Yasuda (2010) show that buyout firms scale their size much faster in reaction to past positive returns than VC funds, which is consistent with assuming that the effort of buyout firms is more scalable than that of VC funds (Corollary 2, part 3). Scalability of effort may also change based on the focus of a fund's investment strategy. For instance, funds that invest in larger companies or other funds should be more scalable compared to funds that invest in early stage companies.

We also explain why private equity returns and size are positively correlated in the cross section. Corollary 2, part 4, predicts that managers that experience a positive shock will raise larger funds and provide higher returns to investors. This is consistent with Kaplan and Schoar's (2005) pooled regression findings, which show that fund size is positively correlated with fund returns when fixed effects for private equity firms are not included.

Our other predictions could be tested using cross-sectional differences across funds as proxies for the model parameters discussed above. For instance, the importance of a manager's ability to add value could vary between funds that invest in early rounds versus later rounds

[15] An exception may be leveraged buyout funds that keep the management of the target firm in place.

(Chemmanur, Krishnan, and Nandy, 2009), between VC funds that take lead versus nonlead positions, and between funds of funds and funds that invest directly in firms. A manager's ability to manipulate entrepreneurs' beliefs may depend on fund characteristics or environment that could affect the matching process, such as geographic separation (Chen *et al.*, 2009), specialization (Gompers, Kovner, and Lerner, 2009; Fulghieri and Sevilir, 2009), or the availability of capital over time.

6. Model Extensions

In this section we discuss extensions to our basic model. We show that our results on performance persistence are fairly general. We also show that our approach is consistent with some of the more recent empirical findings on overall private equity returns.

6.1 AGGREGATE RETURNS TO INVESTING IN PRIVATE EQUITY

Kaplan and Schoar (2005) find that average fund returns in private equity after fees approximately equal the S&P 500 returns. More recently, Phalippou and Gottschalg (2009) find that aggregate average returns of investing in private equity are negative 6% after correcting for risk, weighting by the present value of investment and adjusting for biases in accounting based reporting and sample. This is consistent with the lower returns documented for entrepreneurial investment but is puzzling given the evidence on performance persistence and the increasing amount of investments in private equity.

In our model, expected returns are positive because investors are only willing to participate when the returns to their investments are nonnegative. As we show next, our explanation for performance persistence is consistent with zero or negative aggregate returns in some circumstances if we modify slightly the participation constraint of investors and allow them to view fund investments as long-term propositions, as suggested by Phalippou and Gottschalg (2009). The formal setup and results are provided in the Appendix. Here, we simply sketch the argument.

To see how this argument works, consider a model with just three periods, and suppose that investing in a manager's early funds gives those investors the implicit right to participate in his later funds. This right clearly makes the participation constraint of investors intertemporal since investors recognize that they do not have to break even each period, but rather only across all periods. Now consider fund investors that are evaluating the fund of a manager with no track record and for whom $E[X]$ is zero or close to zero.

If investors decide to invest they will observe first period returns, update their beliefs about the manager's ability, and decide whether they want to invest in the second fund of the same manager. This process repeats again for a third period, after which the manager retires. Having a finite horizon simplifies the problem and makes it easy to communicate the intuition behind the results.

We begin with the third and final period. In this period, it is clear that the manager captures all the surplus since this is the final period and there is no reason to manipulate the beliefs of entrepreneurs. In the second period, however, a manager with positive expected ability has an incentive to manipulate the beliefs of entrepreneurs, as in our main model, and will provide positive expected abnormal returns. Therefore, overall expected returns for fund investors will be positive in the second period, given that managers that are viewed as having negative ability will not be able to raise funds. Finally, in the first period, assuming the participation constraint only needs to be satisfied intertemporally, the positive expected returns in the second period imply that investors may be willing to invest a small amount in the first period even if expected returns are negative. This is because investing the first time around gives them the implicit right to invest in the second fund of the manager if he turns out to have a positive ability. Therefore, the realized returns from investing in a manager's initial fund could be negative—balanced by a positive expected return in subsequent periods as investors learn better which fund managers are likely to have ability and which are not.

The results described above imply that aggregate returns for the entire private equity industry could also be negative, especially during times when there are many new managers entering the industry. Note that there will still be predictability in performance in the cross section: managers with a higher true ability will provide higher returns on average both in the first and in the second time periods. Overall, therefore, our explanation for performance persistence holds under various assumptions about investors' participation constraint, allowing us to explain performance persistence in conjunction with low or negative aggregate returns to investing in private equity.

6.2 PERFORMANCE FEES

In practice, most funds charge a 20–25% carry interest that is, "variable" fee (Gompers and Lerner, 1999; Litvak, 2009; Phalippou and Gottschalg, 2009), in addition to fixed fees. To be consistent with much of the existing literature, up to now we have considered a simplified setting in which the manager charges only a fixed fee. We now show that our results continue to hold when we allow for both fixed and variable fees, so that the addition of a variable fee does not affect whether or not a fund manager decides to leave rents on the table for investors.

We define the variable fee so that the manager's period t payoff also includes a component $v_t(P_tW_t - f_t)$, with $v_t \in [0, 1]$, if the realization of the fund's abnormal returns is positive, and zero otherwise. In other words, the variable fee is a percentage of the fund's realized return, with an implicit option-like characteristic that is similar to how simple carry interest is applied in practice. The rest of the model is as before. For given fees v_t and f_t, the manager's second-stage maximization problem (after the realization of ε_t^P) is now

$$\max_{Q_t, e_t} E_t\left[Q_t f_t - C_t + V_{t+1}(E_{t+1}[X]) + v_t Q_t \int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X)\right], \quad (11)$$

where $\varepsilon_t^{X,0}$ is the realization of ε_t^X that makes the fund's net return to investors equal to zero. Since fund investors are willing to invest as long as their expected return is nonnegative, this optimization problem is subject to

$$E_t\left[\int^{\varepsilon_t^{X,0}} (P_t W_t - f_t) d(\varepsilon^X) + (1 - v_t) \int_{\varepsilon_t^{X,0}} (P_t W_t - f_t) dN(\varepsilon_t^X)\right] \geq 0, \quad (12)$$

where, as usual, the expectation is with respect to the beliefs about managerial ability.

An immediate observation is that the variable fee will not be set equal to 1, that is, 100% of returns, since such a fee structure could never satisfy the participation constraint of investors and would thus lead to a zero return to the fund manager. This is because investors would lose money when the realized fund returns are negative, while receiving a return of zero when fund returns are positive.

The variable fee encourages the manager to exert a higher effort in equilibrium. Nevertheless, our results go through in a similar fashion as above. The manager still has an incentive to manipulate the beliefs of entrepreneurs and he limits the fund's size and provides a positive expected abnormal return to investors, generating performance persistence over time. We summarize this in the following result.

Proposition 4

It is not optimal for the manager to set the variable fee $v^* = 1$. For any given $f_t < \infty$ and $v_t < 1$, there is a value $\varepsilon_t^{P,low}$ such that for realizations $\varepsilon_t^P > \varepsilon_t^{P,low}$, *fund investors' expected abnormal return* $E_t[\alpha_t|\varepsilon_t^P]$ *is strictly positive and zero otherwise. Therefore, at the optimal fee structure* (f_t^*, v_t^*) *fund investors' expected return,* $E_t[\alpha_t]$*, is strictly positive.*

6.3 MANAGERIAL EFFORT TO ADD VALUE

So far, we have assumed that managers have different abilities X that determine fund returns in addition to their effort to select good firms. However, it is natural to think of a manager's effort as adding value in other ways as well. In this section, we show that our results are robust to altering the nature of the manager's effort. In particular, we consider the possibility that the manager's effort adds value directly, rather than through better matching. We also assume that managers' marginal costs of effort vary with their personal abilities.

To study this issue, we modify slightly the expression for a manager's value added to incorporate the assumption that managerial effort leads to greater value added, rather than coming from the manager's "ability", so that $W_t = e_t^X + \varepsilon_t^X - S(Q_t)$, where e_t^X is the manager's effort. Also, define the cost of effort to be C^X, with the marginal cost of effort, $\frac{\partial C^X}{\partial e_t^X}$, increasing in effort e_t^X, decreasing in X, and increasing in fund size Q_t. Managers with lower (marginal) costs of effort are expected to exert a higher equilibrium level of effort. As a result, entrepreneurs prefer to match with managers that have a higher $E[X]$. We keep the other features of the model as before. In the Appendix, we show that when effort is not observed by entrepreneurs, the manager has an incentive to manipulate the beliefs of entrepreneurs by exerting higher effort to add value. As a result, the manager limits the fund's size and, in expectation, does not capture all the value he generates. Therefore, as before, he provides positive expected returns and performance persistence to investors.

7. Conclusion

In this article, we offer an explanation for certain seemingly anomalous patterns of private equity fund returns. Anecdotal evidence suggests that many successful private equity funds are oversubscribed, and that they appear to generate persistent abnormal returns for their investors, in contrast to mutual funds that exhibit little or no performance persistence. We argue that private equity funds are fundamentally different from mutual funds for two reasons: first, two-sided matching plays an important role as private equity funds are eager to match with good firms, while firms try to match with higher ability managers. Second, there is greater asymmetry of information regarding private equity fund managers' ability to add value because observed returns are a function of both selection effort and value-adding ability. Therefore, private equity fund managers are strongly motivated to manipulate the beliefs of firms about their ability to add value, that is, to engage in "signal jamming" of entrepreneurs' beliefs. In particular, by exerting effort to select better firms a manager tries to improve the beliefs of prospective firms about his ability. Managers also keep fund size small because it is less costly to improve the quality of firms in a smaller portfolio.

In equilibrium, firms are not fooled and they correctly form an unbiased expectation. As a result, managers do not benefit from their signal-jamming, although fund investors are made better off. Our model not only explains differences in performance persistence between mutual and private equity funds but also provides new predictions about how our results would vary with the cross-sectional differences among funds and over time.

Our intuition may also apply to different settings. For instance, signal-jamming toward investors instead of entrepreneurs could be important in other settings like hedge funds, where managers may manipulate portfolio risk to provide higher returns and attract investors. This may help explain the performance persistence that has been documented in hedge funds (Jaganathan, Malakhov, and Novikov, 2010). Similarly, investment banks may try to affect their clients' perceptions of their expertise. If clients pay particular attention to past deal volume, banks would try to inflate volume through providing benefits to the issuer that are not entirely transparent to outsiders. This could take the form of guaranteeing analyst coverage, market-making after the IPO, or cutting fees. In other words, bundling of services could create an opportunity to signal jam and attract better deal flow in the future.

Our findings also have interesting implications about the role of information asymmetry in positive assortative matching. In our model, information asymmetry results in excessive effort. However, this could be socially beneficial given that greater search effort is likely to result in better matching between higher quality firms and managers, increasing the success of entrepreneurial firms. Hence, we speculate that policies that require additional disclosure by financial service providers may not necessarily be socially desirable if they lead, for instance, to lower selection effort and poorer matching.

Appendix

Proof of Proposition 1. The Lagrangian for the fund manager's optimization problem at time t, for a given $f_t > 0$, after the realization of ε_t^P is:

$$\max_{Q_t, e_t} \mathcal{L}^2 = Q_t f_t - C_t + \lambda_t (P_t E_t[W_t] - f_t). \tag{13}$$

The Kuhn–Tucker conditions are as follows:

$$f_t - \frac{\partial C}{\partial Q_t} - \lambda_t P_t \frac{\partial S}{\partial Q_t} = 0, \tag{14}$$

$$-\frac{\partial C}{\partial e_t} + \lambda_t E[W_t] \frac{\partial P_t}{\partial e_t} = 0, \tag{15}$$

$$\lambda_t (P_t E[W_t] - f_t) = 0. \tag{16}$$

The proof is based on showing that $\lambda_t > 0$. Let us begin by supposing that $\lambda_t = 0$. In this case the first order condition (FOC) with respect to effort, (15), can only be satisfied if effort e_t is equal to zero. However, the FOC with respect to quantity, (14), cannot be satisfied for $f_t > 0$ given that $\frac{\partial C}{\partial Q_t} = 0$ (from our assumptions regarding the cost function) when effort is equal to zero. Therefore, the solution must have $\lambda_t > 0$, which implies that $P_t E[W_t] - f_t = 0$.

There may also be a region where the participation constraint of investors cannot be satisfied for very low realizations of ε_t^P. In that case, the fund cannot be established in period t and investor returns are again equal to zero. $\square$

Optimal fees: We now analyze the choice of fees f_t that are set at the start of the period. The Lagrangian for the fund manager's optimization problem with respect to the fee is:

$$\max_{f_t} \mathcal{L}^1 = E[Q_t f_t - C_t], \tag{17}$$

subject to Q_t and e_t being defined from (14) and (15), respectively, as well as investors' participation constraint. Note that, from Proposition 1, it may not always be possible to satisfy the investors' participation constraint, so that the region where a fund cannot be established is a function of f_t. Using Leibniz's rule, the FOC can be written as

$$\begin{aligned} \frac{d\mathcal{L}^1}{df_t} = & -\frac{\partial \underline{\varepsilon}_t^P}{\partial f_t} (Q_t f_t - C_t)_{\underline{\varepsilon}_t^P} \\ & + \int_{\underline{\varepsilon}_t^P} \left(\frac{\partial [Q_t f_t - C_t]}{\partial f_t} + \frac{\partial [Q_t f_t - C_t]}{\partial Q_t} \frac{dQ_t}{df_t} + \frac{\partial [Q_t f_t - C_t]}{\partial e_t} \frac{de_t}{df_t} \right) dN(\varepsilon^P) = 0. \end{aligned} \tag{18}$$

When the fund is not established, that is, $\varepsilon_t^P < \underline{\varepsilon}_t^P$. We must have $(Q_t f_t - C_t)_{\underline{\varepsilon}_t^P}$ equal to zero, as otherwise the manager could create slack in the participation constraint of investors either by decreasing fund size or increasing effort (effort is observable in this scenario). In the region where the fund is established we know that $\frac{\partial[Q_t f_t - C_t]}{\partial Q_t} = \frac{\partial \mathcal{L}^2}{\partial Q_t} + \lambda_t P_t \frac{\partial S}{\partial Q_t} = \lambda_t P_t \frac{\partial S}{\partial Q_t}$ since $\frac{\partial \mathcal{L}^2}{\partial Q_t} = 0$ from the envelope theorem (since the Q_t is optimally chosen as per Proposition 1 sometime after the realization of ε_t^P). Likewise, $\frac{\partial[Q_t f_t - C_t]}{\partial e_t} = \frac{\partial \mathcal{L}^2}{\partial e_t} - \lambda_t E[W_t] \frac{\partial P}{\partial e_t} = -\lambda_t E[W_t] \frac{\partial P}{\partial e_t}$ since $\frac{\partial \mathcal{L}^2}{\partial e_t} = 0$. We can therefore rewrite the FOC as

$$\int_{\underline{\varepsilon}_t^P} \left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t} \frac{dQ_t}{df_t} - \lambda_t E[W_t] \frac{\partial P}{\partial e_t} \frac{de_t}{df_t} \right) dN(\varepsilon^P) = 0, \tag{19}$$

where the inner expectation is taken with respect to ε_t^X. From (16) we can see that $E\left[\lambda_t \frac{dQ_t}{df_t}\right] < 0$, while from (15) we can determine that $E\left[\lambda_t \frac{de_t}{df_t}\right] > 0$. Therefore, the solution for the optimal fee, f_t^*, trades off larger fees (first term) against a lower fund size (second term), a higher effort to alleviate the participation constraint of investors (third term), and a lower probability of establishing the fund. □

Proof of Lemma 1 and Proposition 2. From (8) and (9), we see that for a given $f_t < \infty$, the Lagrangian for the fund manager's problem is:

$$\mathcal{L}^2 = E_t[Q_t f_t - C_t + V_{t+1}(E_{t+1}[X])] + \lambda_t(P_t E[W_t] - f_t). \tag{20}$$

The first-order conditions are

$$f_t - \frac{\partial C}{\partial Q_t} - \lambda_t P_t \frac{\partial S}{\partial Q_t} = 0, \tag{21}$$

$$-\frac{\partial C}{\partial e_t} + \frac{\partial}{\partial e_t} E_t[V_{t+1}(E_t[X])] = -\frac{\partial C}{\partial e_t} + E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t}\right] = 0, \tag{22}$$

$$\lambda_t(P_t E[W_t] - f_t) = 0. \tag{23}$$

Note that since effort is not observed in this case, the derivative of the participation constraint with respect to effort is zero. Effort is determined by $\frac{\partial C}{\partial e_t} = E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t}\right] > 0$, with the manager balancing the cost of effort against its benefit in improving his payoff in the future. Therefore, the manager always exerts some effort in order to manipulate entrepreneurs' beliefs. A quick check shows that λ_t can be equal to zero for certain parameter values, and in particular for large values of ε_t^P. When $\lambda_t = 0$, the investors' participation constraint does not bind and fund size Q_t is determined by $f_t = \frac{\partial C}{\partial Q_t}$, that is, the manager equalizes the marginal cost of size (at the conjectured effort level) to the fixed fee. Moreover, since neither (21) nor (22) are functions of ε_t^P when $\lambda_t = 0$, the solution to these two equations defines a minimum level of effort for the fund manager, and a maximum fund size. To see this, note that when $\lambda_t > 0$, the manager selects a smaller Q_t compared to the case when $\lambda_t = 0$ given that $P_t \frac{\partial S}{\partial Q_t} > 0$. We therefore define $Q_t^{\max}$ as this maximal fund size. Likewise, define $e_t^{\min}$ as the equilibrium level of effort at the maximal fund size $Q_t^{\max}$, and note that for values of $Q_t < Q_t^{\max}$, the optimal level of effort will be no less than $e_t^{\min}$ from (22). This establishes Lemma 1.

To show that the equilibrium is unique, we only need to establish that the triple (e_t, e_t^c, Q_t) derived as solutions to the FOCs above along with the equilibrium condition $e_t^c = e_t$, is unique. First, note that since (20) is a single-agent maximization problem with constraints that bound the space of possible solutions, a solution exists and is generically unique for every given entrepreneurs' beliefs about effort e_t^c. Note also (22), which determines optimal effort, can be written as

$$-\frac{\partial C}{\partial e_t} + E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]}(1 - w_t)(X + \varepsilon_t^X - S(Q_t))\frac{1}{P_t(h_t, e_t^c)}\frac{\partial P_t(h_t, e_t)}{\partial e_t}\right] = 0. \quad (24)$$

At equilibrium, under consistent beliefs, we must have $e_t^c = e_t$. Now consider a deviation where entrepreneurs conjecture a higher level of effort. Since $P_t(h_t, e_t^c)$ represents entrepreneurs' beliefs about the average quality of firms in the fund's portfolio, given the conjectured effort level e_t^c, it is increasing in e_t^c. This implies that $\frac{1}{P_t(h_t, e_t^c)}$ is decreasing in e_t^c, so that the positive term in (24) smaller, thus yielding a solution that is still positive but smaller. However, this could not be an equilibrium since the beliefs would not be consistent given we started assuming that entrepreneurs conjecture a higher level of effort. A similar argument shows that a lower level of effort is also not consistent with equilibrium. Hence, the equilibrium is unique.

Investors' expected return after observing the realization of ε_t^P is:

$$E_t[\alpha_t | \varepsilon_t^P] = (z(\overline{P}, e_t, E_t[X]) + \varepsilon_t^P)E[W_t] - f_t, \quad (25)$$

which will be positive when $\varepsilon_t^P > \frac{f_t}{E[W_t(Q_t)]} - z(e_t)$. The maximum of the right-hand side will be achieved when $Q_t = Q_t^{\max}$ and $e_t = e_t^{\min}$ given that $E[W_t(Q_t)]$ is decreasing in Q_t and $z(e_t)$ is increasing in effort. Therefore, if $\varepsilon_t^P > \varepsilon_t^{P,\text{low}} \equiv \frac{f_t}{E[W_t(Q_t^{\max})]} - z(e_t^{\min})$, the expected return must be positive, that is, $E_t[\alpha_t|\varepsilon_t^P] > 0$, because fund size cannot be larger than $Q_t^{\max}$ and the manager's effort cannot be lower than $e_t^{\min}$. Given that ε_t^P is unbounded, for every given f_t there is a realization of ε_t^P such that investors' return $E_t[\alpha_t|\varepsilon_t^P]$ is greater than zero.

Note finally that if $\varepsilon_t^P < \varepsilon_t^{P,low}$, there are two possibilities. First, if investors' participation constraint binds, that is, if $P_tE[W_t] - f_t = 0$, for $Q_t < Q_t^{\max}$, this implies that $f_t > \frac{\partial C}{\partial Q_t}$ and therefore that $\lambda_t > 0$. The other possibility is that for very low realizations of ε_t^P, it will not be possible to satisfy the participation constraint of investors even as fund size approaches zero. In this case the fund will not be established in that period. We define $\underline{\varepsilon}_t^P$ as the minimum realization of ε_t^P such that a fund is established in period t. This proves that expected abnormal return is positive.

This completes the proof for Proposition 2. □

Proof of Proposition 3. The Lagrangian for the fund manager's optimization problem with respect to the fee f_t is

$$\max_{f_t} \mathcal{L}^1 = E_t[Q_tf_t - C_t + V_{t+1}(E_{t+1}[X])], \tag{26}$$

subject to Q_t and e_t being defined from (21) and (22), respectively, as well as investors' participation constraint. Similar to the proof on optimal fees (with symmetric information) above, the FOC can be written as:

$$\begin{aligned}\frac{d\mathcal{L}^1}{df_t} = &-\frac{\partial \underline{\varepsilon}_t^P}{\partial f}(Q_tf_t - C_t + V_{t+1}(E_{t+1}[X]))_{\underline{\varepsilon}_t^P}\\ &+\int_{\underline{\varepsilon}_t^P}\left(\begin{array}{c}\dfrac{\partial E_t[Q_tf_t - C_t + V_{t+1}(E_{t+1}[X])]}{\partial f_t}\\ +\dfrac{\partial E_t[Q_tf_t - C_t + V_{t+1}(E_{t+1}[X])]}{\partial Q_t}\dfrac{dQ_t}{df_t} + \dfrac{\partial E_t[Q_tf_t - C_t + V_{t+1}(E_{t+1}[X])]}{\partial e_t}\dfrac{de_t}{df_t}\end{array}\right)\\ &\times dN(\varepsilon_t^P) = 0.\end{aligned} \tag{27}$$

However, as in the proof on optimal fees (with symmetric information), we know that $(Q_tf_t - C_t + V_{t+1}(E_{t+1}[X]))_{\underline{\varepsilon}_t^P}$ must be equal to zero, and $\frac{\partial[Q_tf_t - C_t]}{\partial Q_t} = \frac{\partial \mathcal{L}^2}{\partial Q_t} + \lambda_t P_t \frac{\partial S}{\partial Q_t} = \lambda_t P_t \frac{\partial S}{\partial Q_t}$ since $\frac{\partial \mathcal{L}^2}{\partial Q_t} = 0$ from the envelope theorem.

Moreover, $\frac{\partial E_t[Q_t f_t - C_t + V_{t+1}(E_{t+1}[X])]}{\partial e_t} = \frac{\partial \mathcal{L}^2}{\partial e_t} = 0$. We can therefore rewrite the FOC as

$$\int_{\underline{\varepsilon}_t^P} \left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t} \frac{dQ_t}{df_t} \right) dN(\varepsilon_t^P) = 0. \quad (28)$$

We can see that $E\left[\lambda_t \frac{dQ_t}{df_t}\right] < 0$ because from the second-stage problem we know that when investors' participation constraint binds $\lambda_t \frac{dQ_t}{df_t} < 0$, and otherwise $\lambda_t \frac{dQ_t}{df_t} = 0$. Therefore (28) defines a solution for the optimal fee, f_t^A, such that the manager trades off higher fees against a lower expected fund size and a lower probability of establishing the fund.

Since the manager solves the same problem each period and market participants learn slowly about the manager's talent X, this generates positive performance persistence across time. We now argue that there is performance persistence or predictability in the cross section as well. In other words, on average managers who have done relatively better in the past will continue to do relatively better in the future. We do this by showing that e_t and $E[\alpha_t]$ are increasing in the expected ability of the manager. First, it is straightforward to show that optimal effort e_t is increasing in $E_t[X]$. To see this, note that from the first-order condition for effort, (22), optimal effort will be higher for higher values of $E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t}\right]$ for every Q_t and f_t. Given that $E_{t+1}[X] = w_t E_t[X] + (1-w_t)\left(\frac{P_t(h_t, e_t)(X + \varepsilon_t^X - S(Q_t))}{P_t(h_t, e_t^c)} + S(Q_t)\right)$ from (7), $E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t}\right]$ is increasing in $E_t[X]$ since the expectation multiplies effort in the term $P_t W_t$, where P_t increases with effort and $E_t[W_t]$ is linearly increasing in $E_t[X]$.

For fund size Q_t, note that the equilibrium value of Q_t obtained from (21) is decreasing in e_t for every f_t. This proves that $E_t[X] - S(Q_t)$ increases as $E_t[X]$ increases, so entrepreneurs would like to match with the manager with the highest expected ability to add value, justifying our earlier assumption. In addition, this implies that $Q_t^{\max}$ decreases and $e_t^{\min}$ increases as $E_t[X]$ increases. This further implies that, for every f_t, the cutoff value $\varepsilon_t^{P,\text{low}} = \frac{f_t}{E[W_t(Q_t^{\max})]} - z(e_t^{\min})$ must decrease as $E_t[X]$ increases.

We now characterize what happens to the fee f_t. To find the sign of $\frac{df_t}{dE[X|h_{t-1}]}$, we take the total derivative of $\frac{\partial \mathcal{L}^1}{\partial f}$ with respect to $E[X|h_{t-1}]$:

$$\frac{\partial^2 \mathcal{L}^1}{\partial f_t^2} \frac{df_t}{dE_t[X]} + \frac{\partial^2 \mathcal{L}^1}{\partial f_t \partial E_t[X]} = 0. \quad (29)$$

Since $\frac{\partial^2\mathcal{L}^1}{\partial f_t^2} < 0$ from the SOC, we have $\text{sign}\left(\frac{df_t}{dE_t[X]}\right) = \text{sign}\left(\frac{\partial^2\mathcal{L}^1}{\partial f_t \partial E_t[X]}\right)$. This latter expression can be written as

$$\frac{\partial^2\mathcal{L}^1}{\partial f_t E_t[X]} = -\frac{\partial \underline{\varepsilon}_t^P}{E_t[X]}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} + \frac{\partial}{E_t[X]}\int_{\underline{\varepsilon}_t^P}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right) dN(\varepsilon_t^P). \tag{30}$$

Note that $\frac{\partial \underline{\varepsilon}_t^P}{E_t[X]} < 0$, which can be seen by solving for $\underline{\varepsilon}_t^P$ from investors' participation constraint: higher $E_t[X]$ implies higher effort, which results in lower $\underline{\varepsilon}_t^P$. On the other hand, the term $\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P}$ must be negative since for large realizations of ε_t^P the term $Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}$ is positive since investors' participation constraint does not bind ($\lambda_t = 0$) and Q_t is positive. Therefore, in order for (28) to be satisfied for lower realizations of ε_t^P it must be that $(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t})_{\underline{\varepsilon}_t^P} < 0$. As a result, $-\frac{\partial \underline{\varepsilon}_t^P}{\partial w_t}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} < 0$. The second term above is zero since $\int_{\underline{\varepsilon}_t^P}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right) dN(\varepsilon_t^P)$ is identically equal to zero at the equilibrium from (28), and Q_t is not a direct function of $E_t[X]$. We can therefore conclude that

$$\frac{\partial^2\mathcal{L}^1}{\partial f_t \partial E_t[X]} = -\frac{\partial \underline{\varepsilon}_t^P}{\partial E_t[X]}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} < 0. \tag{31}$$

Hence, $\frac{df}{dE_t[X]} < 0$ and the fund manager will choose a lower fee when $E_t[X]$ is larger. Therefore, $\varepsilon_t^{P,\text{low}}$ is decreasing in $E_t[X]$ and investors' expected return increases. □

Proof of Corollary 1. It is straightforward to show that optimal effort e_t decreases as w_t increases. To see this, note that $\frac{\partial E_{t+1}[X]}{\partial e_t}$ is decreasing in w_t from the definition of $E_{t+1}[X]$ in (7). Therefore, for every Q_t and f_t, the FOC for effort, Equation (22), will yield a lower value for e_t as w_t increases. For fund size Q_t, note that for $\lambda_t = 0$, the equilibrium value of Q_t obtained from (21) is decreasing in e_t for every f_t. Put together, this implies that $Q_t^{\max}$ increases and $e_t^{\min}$ decreases as w_t increases. This implies that, for every f_t, the cutoff value $\varepsilon_t^{P,\text{low}} = \frac{f_t}{E[W_t(Q_t^{\max})]} - z(e_t^{\min})$ must increase as w_t increases.

We now characterize what happens to the fee f_t. To find the sign of $\frac{df_t}{dw_t}$, we take the total derivative of $\frac{\partial \mathcal{L}^1}{\partial f}$ with respect to w_t:

$$\frac{\partial^2 \mathcal{L}^1}{\partial f_t^2}\frac{df_t}{dw_t} + \frac{\partial^2 \mathcal{L}^1}{\partial f_t \partial w_t} = 0. \tag{32}$$

Since $\frac{\partial^2 \mathcal{L}^1}{\partial f_t^2} < 0$ from the SOC, we have $\mathrm{sign}\left(\frac{df_t}{dw_t}\right) = \mathrm{sign}\left(\frac{\partial^2 \mathcal{L}^1}{\partial f_t \partial w_t}\right)$. This latter expression can be written as

$$\frac{\partial^2 \mathcal{L}^1}{\partial f_t \partial w_t} = -\frac{\partial \underline{\varepsilon}_t^P}{\partial w_t}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} + \frac{\partial}{\partial w_t}\int_{\underline{\varepsilon}_t^P}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)dN(\varepsilon_t^P). \tag{33}$$

Note that $\frac{\partial \underline{\varepsilon}_t^P}{\partial w_t} > 0$, which can be seen by solving for $\underline{\varepsilon}_t^P$ from investors' participation constraint: higher w_t implies lower effort, which results in higher $\underline{\varepsilon}_t^P$. On the other hand, the term $\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P}$ is negative since for large realizations of ε_t^P, investors' participation constraint does not bind, so that $\lambda_t = 0$ and Q_t is positive. Therefore, for (28) to be satisfied for lower realizations of ε_t^P it must be that $(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}) < 0$. As a result, $-\frac{\partial \underline{\varepsilon}_t^P}{\partial w_t}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} > 0$. The second term above is zero since $\int_{\underline{\varepsilon}_t^P}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)dN(\varepsilon_t^P)$ is identically equal to zero at the equilibrium from the FOC, (28), and none of the terms are direct functions of w. We can therefore conclude that

$$\frac{\partial^2 \mathcal{L}^1}{\partial f_t \partial w_t} = -\frac{\partial \underline{\varepsilon}_t^P}{\partial w_t}\left(Q_t + \lambda_t P_t \frac{\partial S}{\partial Q_t}\frac{dQ_t}{df_t}\right)_{\underline{\varepsilon}_t^P} > 0. \tag{34}$$

Hence, $\frac{df}{dw_t} > 0$ and the fund manager will choose higher fees when w_t is larger. Therefore, $\varepsilon_t^{P,\text{low}}$ is increasing in w_t and the probability of fund investors' having a positive expected return decreases. □

Proof of Corollary 2. In Corollary1 we provide the proof with respect to changes in w_t. The proofs of all results in Corollary 2 are similar as they rely on the same intuition as in Corollary1 and are therefore omitted. □

Proof of Proposition 4. From the participation constraint it is obvious that v_t must be strictly less than 1, as otherwise investors' participation constraint could not be satisfied. The Kuhn Tucker conditions are

$$f_t - \frac{\partial C_t}{\partial Q_t} + v_t \int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) + Q_t v_t \frac{\partial}{\partial Q_t}\left(\int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) \right)$$
$$+\lambda_t \frac{\partial}{\partial Q_t}\left(\int^{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) + (1 - v_t) \int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) \right) = 0, \tag{35}$$

$$-\frac{\partial C_t}{\partial e_t} + E_t\left[\frac{\partial V_t}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t} \right] + v_t Q_t \int_{\varepsilon_t^{X,0}} \frac{\partial P_t}{\partial e_t} E_t[W_t] dN(\varepsilon_t^X)$$
$$+\lambda_t \frac{\partial}{\partial e_t}\left(\begin{array}{c} \int^{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) \\ +(1 - v_t) \int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) \end{array} \right) = 0, \tag{36}$$

$$\lambda_t \left(\int^{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) + (1 - v_t) \int_{\varepsilon_t^{X,0}} (P_t E_t[W_t] - f_t) dN(\varepsilon_t^X) \right) = 0. \tag{37}$$

Note that $\lambda_t = 0$ can be a potential solution. When $\lambda_t = 0$, the maximum fund size $Q_t^{\max}$ is determined by $f_t - \frac{\partial C}{\partial Q_t} + v_t \int_{\varepsilon_t^{X,0}} (P_t E[W_t] - f_t) dN(\varepsilon_t^X) + Q_t v_t \frac{\partial}{\partial Q_t}\left(\int_{\varepsilon_t^{X,0}} (P_t E[W_t] - f_t) dN(\varepsilon_t^X) \right)$, while the minimum level of effort $e_t^{\min}$ is determined from $-\frac{\partial C}{\partial e_t} + E_t\left[\frac{\partial V_t}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t} \right] + v_t Q_t \int_{\varepsilon_t^{X,0}} \frac{\partial P}{\partial e_t} E_t[W_t] dN(\varepsilon_t^X)$. As before, by substituting $Q_t^{\max}$ and $e_t^{\min}$ in the investors' participation constraint one can derive a cutoff value $\varepsilon_t^{P,\text{low}}$ for any f_t and v_t such that investors' expected abnormal return is positive. If this is true for any f_t and v_t, it will be true for the optimal fees f_t^* and v_t^* such that $f_t^* < \infty$ and $v_t^* < 1$. We have argued above that $v_t^* < 1$ and f_t^* is bounded from the first-stage optimization problem because, as before, larger f_t implies a smaller fund size and a lower probability of establishing the fund. □

Results from a finite horizon model (Section 6.1). Here we offer a simplified version of the model where: (i) investors who put money into a fund are given the right of first refusal for the next fund the manager forms. This implies that investors should view their participation constraint as intertemporal rather than period by period. (ii) The horizon T is finite (rather than infinite) and for simplicity equal to 3. Putting these two assumptions together implies that investors' participation constraint for investing in a particular fund can be written as

$$E_t\left[\sum_{i=t}^{3} \Pr\,(E_i[X] > 0)\;\alpha_i|\varepsilon_i^P\right] = E_t\left[\sum_{i=t}^{3} \Pr\,(E_i[X] > 0)(P_i W_i - f_i)|\varepsilon_i^P\right] \geq 0, \tag{38}$$

for $t \in \{1, 2, 3\}$.

We can solve this model by backward induction. Consider the fund manager's problem at $t=3$, assuming $E[X|h_2] > 0$:

$$\begin{aligned}\mathcal{L}_T^2 &= Q_3 f_3 - C_3 + V_4(E[X|h_3]) + \lambda_3(P_3 E[W_3] - f_3)\\ &= Q_3 f_3 - C_3 + \lambda_3(P_3 E[W_3] - f_3)\end{aligned} \tag{39}$$

since, given $t=3$ is the final period, $V_4 = 0$. The FOCs are

$$f_3 - \frac{\partial C}{\partial Q_3} - \lambda_3 P_3 \frac{\partial S}{\partial Q_3} = 0 \tag{40}$$

$$-\frac{\partial C}{\partial e_3} = 0 \tag{41}$$

$$\lambda_3(P_3 E[W_3] - f_3) = 0. \tag{42}$$

It is clear that (41) can only be satisfied at $e_3 = 0$. An argument similar to the one for the symmetric information case establishes that $\lambda_3 > 0$ and that $P_3 E[W_3] - f_3 = 0$, so that investors earn no abnormal returns in the final period.

We can now solve the manager's problem at $t=2$:

$$\begin{aligned}\mathcal{L}_2^2 &= Q_2 f_2 - C_2 + V_3(E[X|h_2]) + \lambda_2\\ &\quad\times((P_2 E[W_2] - f_2)Q_2 + E[\Pr\,(E[X|h_2] > 0)(P_3 W_3 - f_3)Q_3])\\ &= Q_2 f_2 - C_2 + V_3(E[X|h_2]) + \lambda_2(P_2 E[W_2] - f_2)Q_2\end{aligned} \tag{43}$$

since $E[P_3 W_3 - f_3] = 0$. The FOCs are

$$f_2 - \frac{\partial C}{\partial Q_2} - \lambda_2 P_2 \frac{\partial S}{\partial Q_2} = 0 \tag{44}$$

$$-\frac{\partial C}{\partial e_2} + \frac{\partial V_3}{\partial E[X|h_2]}\frac{\partial E[X|h_2]}{\partial e_2} = 0 \tag{45}$$

$$\lambda_2(P_2 E[W_2] - f_2)Q_2 = 0. \tag{46}$$

This problem is exactly like the infinite horizon model we studied previously. We can thus conclude that for given f_2, $E_2[\alpha_2|\varepsilon_2^P]$ is either positive for ε_2^P large enough, or zero. Therefore, we also have that $E_2[\alpha_2] > 0$.

Now consider the first period problem for the manager:

$$\begin{aligned}\mathcal{L}_1^2 &= Q_1 f_1 - C_1 + V_2(E[X|h_1]) \\ &\quad + \lambda_1\Big((P_1 E[W_1] - f_1)Q_1 + E[\Pr(E[X|h_1] > 0)(P_2 W_2 - f_2)Q_2] \\ &\quad + E[\Pr(E[X|h_2] > 0)] \times (P_3 W_3 - f_3)Q_3\Big) \\ &= Q_1 f_1 - C_1 + V_2(E[X|h_1]) + \lambda_1\Big((P_1 E[W_1] - f_1)Q_1 + E[\Pr(E[X|h_1] > 0) \\ &\quad \times (P_2 W_2 - f_2)Q_2]\Big),\end{aligned} \tag{47}$$

again since $E[P_3 W_3 - f_3] = 0$. The FOCs are

$$f_1 - \frac{\partial C}{\partial Q_1} - \lambda_1\left(Q_1 P_1 \frac{\partial S}{\partial Q_1} + (P_1 E[W_1] - f_1) + \frac{\partial}{\partial Q_1} E[\Pr(E[X|h_1] > 0)(P_2 W_2 - f_2)Q_2]\right) = 0, \tag{48}$$

$$-\frac{\partial C}{\partial e_1} + \frac{\partial V_2}{\partial E[X|h_1]}\frac{\partial E[X|h_1]}{\partial e_1} = 0, \tag{49}$$

$$\lambda_1((P_1 E[W_1] - f_1)Q_1 + E[\Pr(E[X|h_1] > 0)(P_2 W_2 - f_2)Q_2]) = 0. \tag{50}$$

Note that, as before, (49) yields a positive level of effort in equilibrium: $e_1 > 0$. Suppose now that $\lambda_1 = 0$. Then $f_1 - \frac{\partial C}{\partial Q_1} = 0$ will define the maximum fund size, $Q_1^{\max}$, that the fund manager would wish to operate.

For $\lambda_1 > 0$, the fund size will be smaller. Note, however, that when $\lambda_1 > 0$, we must have:

$$((P_1 E[W_1] - f_1)Q_1 + E[\Pr(E[X|h_1] > 0)(P_2 W_2 - f_2)Q_2]) = 0, \tag{51}$$

which implies that $P_1 E[W_1] - f_1 < 0$ since $E[P_2 W_2 - f_2] > 0$. In other words, for the cases where investors' participation constraint binds intertemporally, the first period return must be negative. For $\lambda_1 = 0$, it is straightforward to see that for some parameter values $P_1 E[W_1] - f_1 > 0$, while for others $P_1 E[W_1] - f_1 < 0$ is consistent with satisfying investors' participation constraint, for given f_1 and $E[f_2]$.

Finally, note that to find the optimal fee, the manager maximizes

$$\begin{aligned} \max_{f_t} \mathcal{L}_1^1 = E[Q_1 f_1 - C_1 + V_2(E[X|h_1]) \\ + \lambda_1((P_1 E[W_1] - f_1)Q_1 + E[\Pr(E[X|h_1] > 0)(P_2 W_2 - f_2)Q_2])], \end{aligned} \tag{52}$$

subject to Q_1 and e_1 being optimally chosen as per the solution above. Since, for fixed beliefs $E[X]$ about managerial ability we know that $Q_2 > Q_1$ since there is less uncertainty about the manager's ability at time 2 (and, hence, less incentive to manipulate information by keeping the fund size smaller), for any given $E[f_2]$ there must be values of f_1 such that $(P_1 E[W_1] - f_1)Q_1 < 0$, while still satisfying investors' participation constraint. Choosing such a value will be optimal when the second period signal-jamming incentive is large (i.e., when $\frac{\partial V_3}{\partial E[X|h_2]} \frac{\partial E[X|h_2]}{\partial e_2}$ is large) so that $E_2[\Pr(E[X|h_1] > 0)\alpha_2] >> 0$, but the initial average quality of fund managers, $E[X]$, is low (or negative).

The existence of model parameters that yield negative first period returns in equilibrium can easily be illustrated with an example. Consider a manager with $E[X] = 0$ at time zero. This ensures that whenever the manager raises any funds in the first time period, $E[W_1] = E[X] - S(Q_1)$ is negative for any $Q_1 > 0$. Given that the first period fee $f_1 \geq 0$ will make investor returns even more negative and is determined by considering that the fund may not be established at all, the manager in equilibrium chooses an initial fee which ensures the fund will be established with positive probability (otherwise the manager's return would be zero). Therefore, whenever the fund is established, the fund investors' expected returns from the first period is negative. Finally, note that for a small enough but strictly positive Q_1, it will be optimal for investors to providing funding to the manager since the expected return will be strictly positive in the second period given a positive probability that the fund manager actually have ability ($X > 0$) or is lucky and delivers sufficiently positive returns in the first period. □

Proof of results from Section 6.3. The manager's value added is determined by $E[W_t] = e_t^X + \varepsilon_t^X - S(Q_t)$. From (8) and (9), we see that for a given f_t, the Lagrangian for the fund manager's problem is

$$\mathcal{L}^2 = E_t\left[Q_t f_t - C_t^X + V_{t+1}(E_{t+1}[X])\right] + \lambda_t(P_t E[W_t] - f_t). \tag{53}$$

The first-order conditions are

$$f_t - \frac{\partial C^X}{\partial Q_t} - \lambda_t E_t\left[P_t \frac{\partial S}{\partial Q_t}\right] = 0, \tag{54}$$

$$-\frac{\partial C^X}{\partial e_t^X} + E_t\left[\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t^X}\right] = 0, \tag{55}$$

$$\lambda_t(P_t E[W_t] - f_t) = 0. \tag{56}$$

These first-order conditions are similar to those before, except that now managerial effort adds value. The proof is similar to that for Proposition 2. Much as in the case studied in Proposition 2, since effort is not observed the derivative of the participation constraint with respect to effort is zero. From (55) we see that the manager always exerts some effort in order to manipulate entrepreneurs' beliefs. A quick check shows that λ_t can be equal to zero for certain parameter values. When $\lambda_t = 0$, fund size Q_t is determined by $f_t = \frac{\partial C}{\partial Q_t}$, so that the manager equalizes the marginal cost of size (at the conjectured effort level to select firms) to the fixed fee. Again as in the proof of Proposition 2, this is the largest fund size that the manager is willing to operate because when $\lambda_t > 0$ the manager selects a smaller Q_t compared to the case when $\lambda_t = 0$ given that $P_t \frac{\partial S}{\partial Q_t} > 0$, and we define $Q_t^{\max}$ as this maximal fund size. Define as well $e_t^{X\min}$ as the equilibrium level of effort at the maximal fund size $Q_t^{\max}$, defined as the solution to (55), and note that $e_t^{X\min}$ represents the lowest level of effort the fund manager will find it optimal to exert since for values of $Q_t < Q_t^{\max}$, the optimal level of effort will be no less than $e_t^{X\min}$.

Investors' expected return after observing the realization of ε_t^P is $E_t[\alpha_t|\varepsilon_t^P] = (z(\overline{P}, e_t^X) + \varepsilon_t^P)E[W_t] - f_t$. Since, all else equal, a manager's effort to add value is determined by X, we could define $z(\overline{P}, e_t^X) = z(\overline{P}, E[X])$, which will be positive when $\varepsilon_t^P > \frac{f_t}{E[W_t(Q_t)]} - z(e_t^X)$. The maximum of the right-hand side will be achieved when $Q_t = Q_t^{\max}$ and $e_t = e_t^{X\min}$ given that $E[W_t(Q_t)]$ is decreasing in Q_t and $z(e_t^X)$ is increasing in effort and e_t^X does not depend on the realization of ε_t^P. Therefore, if

$\varepsilon_t^P > \varepsilon_t^{P,\text{low}} \equiv \frac{f_t}{E[W_t(Q_t^{\max})]} - z(e_t^{X\min})$, the expected return must be positive, that is, $E_t[\alpha_t|\varepsilon_t^P] > 0$, because fund size will never optimally be larger than $Q_t^{\max}$ and the manager's selection effort will not be chosen to be lower than $e_t^{X\min}$. Given that ε_t^P is unbounded, for every given f_t there are realizations of ε_t^P such that investors' return $E_t[\alpha_t|\varepsilon_t^P]$ is greater than zero.

Note finally that if $\varepsilon_t^P < \varepsilon_t^{P,\text{low}}$, there are two possibilities. First, if investors' participation constraint binds, that is, if $P_t E[W_t] - f_t = 0$ for $Q_t < Q_t^{\max}$, this implies that $f_t > \frac{\partial C}{\partial Q_t}$ and therefore that $\lambda_t > 0$. The other possibility is that for very low realizations of ε_t^P, it will not be possible to satisfy the participation constraint of investors even as fund size approaches zero. In this case the fund will not be established in that period. We define $\underline{\varepsilon}_t^P$ as the minimum realization of ε_t^P such that a fund is established in period t. This proves that expected abnormal return is positive, following the arguments in Proposition 2. □

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