Optimal Monetary and Fiscal Policy in a Currency Union with Frictional Goods Markets[*]

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Abstract

We develop an open economy model of a currency union with frictional goods markets and endogenous search decisions to study optimal monetary and fiscal policy. Households finance consumption with a common currency and can search for locally produced goods across regions that differ in their market characteristics. Equilibrium is generically inefficient due to regional spillovers from endogenous search decisions. While monetary policy alone cannot correct this distortion, fiscal policy can help improve allocations by taxing or subsidizing production at the regional level. When households of only one region can search, optimal policy entails a deviation from the Friedman rule and a production subsidy (tax) if there is underinvestment (overinvestment) in search decisions. Optimal policy when households from both region search requires the Friedman rule and zero production taxes in both regions.

Keywords: currency unions, search frictions, optimal monetary and fiscal policy

JEL Classification Codes: C92, D83, E40

1 Introduction

It is widely thought that being in a currency union matters for trade and welfare. According to conventional wisdom, members of the union share a common currency, thereby lowering costs and promoting trade across regions. However, the size of these trade-stimulating effects are quite varied in the literature (see e.g. Rose and vanWincoop 2001, Santos, Silva, and Tenreyro 2010, Head and Mayer 2014).[1] This is consistent with the view that there are other costs and frictions that being in a currency union cannot ameliorate. For instance, local consumers with tastes for regional variety may face relatively more costly barriers in locating or obtaining foreign-produced goods than local ones. As emphasized by Kneller and Pisu (2011), "identifying the first contact" and "establishing initial dialogue" are more common impediments to cross-border trade than language, cultural, or regulatory differences among regions.

In this paper, we analyze a situation where countries are already in a currency union and investigate the optimal monetary and fiscal policies when there are search externalities. As highlighted in the literature above, information and search barriers distort trade flows and may prevail even when trading in a currency union. We capture the costly and time-consuming nature of trade in a currency union by modeling the decisions of consumers to search for local and foreign produced goods in frictional goods markets.

To explore how these features affect optimal policy design, we develop an open economy model of a currency union with endogenous market tightness that can differ across regions of the union. These features formalize the effects of individual search decisions on the intensive and extensive margins of trade, i.e. quantity per trade and total number of trades. Our benchmark model is an open economy version of Rocheteau and Wright (2005) with two regions that share a common currency and have a single central bank that sets monetary policy. Each region produces a tradable consumption good that can be financed with the common currency. Regional trade occurs in decentralized goods markets where households and producers meet bilaterally and negotiate the terms of trade. While producers have immobile factors of production, households can invest to search for foreign-produced goods.[2] However due to frictions, search by an individual household creates spillovers to other buyers in the currency union. Terms of trade in frictional goods markets are negotiated bilaterally, where regional

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[1]For instance, Head and Mayer (2014) write in their review article: "The trade effects of common currencies have been the subject of controversy. Our mean over 104 estimates is 0.79, which corresponds to a doubling of trade. This is substantially smaller than initial estimates by Rose (2000)... implying more than tripling trade... Baldwin (2006), synthesizing a stream of papers focusing mainly on the Euro, puts the currency effect at about 30%. Santos, Silva, and Tenreyro (2010) find virtually no effects on trade for the Euro, after taking account the high level of trade integration of Eurozone members even before they formed a common currency."

[2]In practice, search for foreign goods can be done in several ways, e.g. by searching on the internet or even by commuting to other countries. In the model, it is not necessary for consumers to physically move to the location where production takes place, which we think is natural given the increasing prevalence of online commerce.

differences in bargaining power and matching efficiencies affect ex-ante search decisions.

A key implication of the model is that the intensity of search for foreign-produced goods is an endogenous response to regional differences in the currency union. In particular, local market tightness decreases with the market power of foreign producers but increase with local mark ups. However, whether or not search is socially efficient is more subtle since an individual household also creates a congestion externality for other agents in the union. Indeed, we show that equilibrium with endogenous market tightness is generically inefficient along both intensive and extensive margins of trade.[3] First, output per trade is inefficiently low due to an asynchronicity between production and the accumulation real balances to finance consumption of foreign produced goods. Second, the number of trades can be inefficiently high or low as agents do not internalize their search decisions. Efficiency jointly requires the Friedman rule at the union level and the Hosios condition at the regional level. Intuitively, the Friedman rule eliminates the cost of holding money across periods which alleviates the distortion on households' intertemporal decisions. Meanwhile, the Hosios condition eliminates the matching inefficiency by providing an appropriate division of the trade surplus.

To highlight these mechanisms, we first study an environment where only households of one region choose to search for foreign goods. Search by households in the other region is exogenously fixed. A unique stationary monetary equilibrium exists if one region has sufficiently lower markups and higher matching efficiency than the other region.[4] In that case, households underinvest (overinvest) when searching for foreign goods if their bargaining power with local producers is larger (smaller) than their contribution to the matching process. In addition, a permanent negative shock to local matching efficiency can make households switch from underinvesting to overinvesting in their search efforts. This arises even at the Friedman rule, except in the knife edge case where the Hosios condition is also satisfied.

Given inefficiencies along both intensive and extensive margins of trade, we next consider monetary and fiscal policies that can alleviate these distortions. Without endogenous search, monetary policy alone is sufficient to achieve the first best since the only distortion is at the intensive margin (output per trade). When endogenous search is possible, an additional policy instrument is needed to correct the extensive margin distortions arising from agents' search decisions (number of trades).[5] We therefore introduce regional fiscal policies that require a tax or subsidy to local producers. If

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[3]Similar types of inefficiencies arise in search models of money with both intensive and extensive margins of trade, as in Rocheteau and Wright (2005) and Berentsen et al. (2007) (see Section 1.1 for a discussion of related literature).

[4]Equilibrium with search is not generically unique due to complementarities between search decisions and production. Intuitively, more production abroad raises the expected surplus for households to relocate which increases search and the hence total number of trades abroad. This raises the gain to produce abroad, thereby further raising production.

[5]Gomis-Porqueras et al. (2013), Lehmann (2012), and Hiraguchi and Kobayashi (2014) also explore monetary and fiscal policies aimed at alleviating the intensive and extensive inefficiencies.

there is underinvestment (overinvestment) in search, a subsidy (tax) to foreign producers increases (decreases) equilibrium search by raising (reducing) output per trade and hence the net gains from searching for the foreign produced good. The optimal policy mix is then a money growth rate and production subsidy or tax that maximizes social welfare for the union.[6]

When household bargaining power is higher than their contribution to the matching process, local households underinvest when searching for foreign goods. Since a household's incentive to search decreases with their bargaining power with local producers, a sufficiently large bargaining power induces households to invest too little in search activities. In that case, the total volume of trade can be inefficiently low. The resulting optimal policy entails a higher money growth rate than the Friedman rule and a subsidy to foreign producers. Intuitively, a deviation from the Friedman rule is optimal since an increase in money growth increases search for foreign goods. This arises when the foreign region has more favorable trading conditions, i.e. households have higher bargaining power or they face higher matching efficiency. However, higher money growth also decreases production in both regions. To counteract this distortion, the policymaker sets a production subsidy in the foreign economy, increasing its production and the total number of matches. On the other hand, when household's bargaining power is too low, local households overinvest when searching abroad. The resulting optimal policy is the Friedman rule and a tax to foreign producers. The policymaker therefore deflates at the rate of time preference to minimize the intensive margin distortion. Such policy in turn has the additional benefit of decreasing inefficiently high search intensity and bringing it closer to its first best level.

When local households from both regions can choose to search abroad, we find that the optimal policy prescription, for our numerical example, is the Friedman rule and no tax or subsidy. Intuitively, this arises since monetary policy is more effective at correcting the intensive margin than fiscal policy is at alleviating the extensive margin. This implies no search abroad is carried out by households of either region.

The paper is organized as follows. Section 1.1 briefly discusses related literature. Section 2 describes the environment. Section 3 studies the social planner problem, and Section 4 characterizes the monetary equilibrium. We consider one sided search in Section 5 and illustrate the optimal monetary-fiscal policy mix for the union. We extend the analysis to include two sided search in Section 6. Section 7 concludes. All proofs can be found in the Appendix.

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[6]Implicit in this analysis is our assumption that the monetary union is also a fiscal union: the government implements regional fiscal policy financed by a common central bank that sets a union wide money growth rate.This resonates with a classic idea from Kenen (1969) on the importance of fiscal integration for a monetary union: "It is a chief function of fiscal policy, using both sides of the budget, to offset or compensate for regional differences, whether in earned income or in unemployment rates. The large-scale transfer payments built into fiscal systems are interregional, not just interpersonal..."

1.1 Related Literature

This paper relates with four broad strands of the literature. The first explores how various features of labor or goods market affect the frequency and output per trade. The second studies optimal monetary and fiscal policies within the Ramsey tradition when search is endogenous. We also contribute to the literature that studies the effects of policy in currency unions. Finally, some of our modeling ingredients are similar to real open economy models that study how government policies and information frictions distort trade flows.

We consider a monetary model and formulate agents' search decision in a similar way as the literature that have considered such decisions in the context of goods or labor markets. See Rocheteau and Nosal (2017) and Lagos et al. (2017) for a recent survey of literature on search theoretic models of money. Lagos and Rocheteau (2005) and Berentsen et al. (2007) show the Friedman rule generates inefficient search decisions while the equilibrium under the Hosios rule generates an inefficiently low quantity of goods in each trade. In models with endogenous mobility of workers across sectors, Chang (2012) and Branch et al. (2016) find that the allocation of workers across sectors is typically inefficient. In contrast to these papers, we consider search across regions that belong to a currency union and analyze optimal monetary and fiscal policy. We show how this generates spillovers across regions that have important implications for policy design and the trade volume across regions.

This paper also contributes to the literature on optimal monetary and fiscal policy in environments with search externalities where the government can commit to their future policies. With search in the labor market, Gomis-Porqueras et al. (2013) show that a production subsidy in frictional goods markets, financed by money creation, and a vacancy subsidy, financed by a dividend tax, achieve efficiency even when the Hosios condition does not hold. In a different environment, Lehmann (2012) shows the optimal money growth rate decreases with the workers' bargaining power, unemployment benefits, and the payroll tax rate. When buyers choose search intensity in the frictional goods market context, Hiraguchi and Kobayashi (2014) show the Friedman rule may not be optimal and fiscal policy is required to achieve efficiency. Finally, Herrenbreuck (2015) studies optimal monetary policy in an open economy model with price posting and shows inflation can have non-monotonic effects due consumers' search intensity.

Our paper also contributes to the literature that studies monetary and fiscal policy in a currency unions. In a flexible price search model, Bignon et al. (2015) show currency integration may magnify default incentives, leading to more stringent credit rationing and lower welfare than in a regime of two currencies. The integration of credit markets restores the optimality of the currency union. As

with the previous authors, we take the existence of a currency union as given.[7] In particular, we study both optimal monetary and fiscal policy in a model where market tightness is endogenous.[8] In contrast to the previous papers, we consider how monetary and fiscal policy can alleviate distortions arising from endogenous market tightness and focus on the positive and normative implications for the intensive and extensive margins of trade.

Finally, some of our model ingredients share similarities with international trade models of real open economies, starting with Rauch (1996), that examines the impact of government policies and various information frictions on trade across regions. We consider a similar formalization of costly search for international partners as in Krolikrowski and McCallum (2017). These authors consider a real model where producers engage in costly search for buyers which leads to an endogenous fraction of unmatched producers. Similar to our paper, they find the equilibrium is socially inefficient where aggregate trade flows can be lower than optimal through price and extensive margin effects.

In contrast with these previous studies, we consider a frictional environment where agents belong to a monetary currency union and highlight the roles for monetary and fiscal policies in alleviating distortions arising from individual search activities. A key contribution of this paper is the analysis of the optimal monetary and fiscal policy mix in a currency union where market tightness is endogenous and generated by individual decisions to search for foreign-produced goods.

2   Environment

Time is discrete and continues forever. There are two regions, $i, j \in \{1, 2\}$, each with a continuum of infinitely lived buyers (or households), denoted by $\mathbb{B}_i$ and sellers (or producers), denoted by $\mathbb{S}_j$. Agents are exogenously assigned to one of the regions. Region 1 has a measure $2n$ of agents and region 2 has a measure 2 of agents, where $n \in (0, 1]$ is the relative size of region 1.[9]

Each period consists of two stages. In the first, agents meet bilaterally in decentralized markets (DM) where buyers want to consume a regional good that only sellers from $j \in \{1, 2\}$ can produce. Let $q_j \in \mathbb{R}_+$ denote the quantity of output produced in region j. Sellers have immobile factors of production and cannot produce the other region's good. There is lack of record keeping, no public

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[7]Zhang (2014) and Gomis-Porqueras et al. (2017) study multi-country environments with multiple currencies and characterize under what conditions a single currency is traded.

[8]There is an extensive literature on monetary and fiscal policy in monetary models in a currency union with nominal rigidities. See e.g. Corsetti and Pesenti (2001), Beestma and Jensen (2005), Gali and Monacelli (2008), and references therein. In these papers, there is a single central bank that sets a common interest rate for the union while fiscal policy is determined at the regional level through the choice of government spending. In response to country specific supply shocks, monetary policy is used to stabilize the union-wide economy while fiscal policy is used to stabilize regional inflation differences and the terms of trade.

[9]There are different interpretations of buyers and sellers, e.g., households purchasing consumption goods from productive firms, or firms acquiring productive capital from suppliers. Here we take the former interpretation though our theory would also apply to the latter.

information nor communication of individual trading histories, and no enforcement. These frictions preclude unsecured credit arrangements, thereby generating a need for a medium of exchange. In the second stage, there is a frictionless centralized market (CM) where all agents can produce and consume a homogeneous and the perishable numéraire good, $x \in \mathbb{R}$, by supplying labor with a linear production technology, $f(y) = y$. At the end of each CM, buyers return to their region of origin. The discount factor for all agents is $\beta = (1 + r)^{-1}$, where $r > 0$ is the rate of time preference.

Search. In the DM, buyers are mobile while sellers are immobile. Buyers have tastes for both domestic and foreign goods while sellers have immobile factors of production. This can be a result of regulation or prohibitively costly legal barriers that differ across regions. At the beginning of each period, a buyer $b \in \mathbb{B}_i$ from region i can invest $\rho_i^b \in [0, 1]$ units of effort to search for foreign-produced goods in region $i' \in \{1, 2\} \neq i$. With complementary probability $1 - \rho_i^b$, the buyer can only search at home. The decision ρ_i^b therefore represents a region i buyer's search intensity in the goods market of region i'. When $\rho_i^b = 0$, the buyer only searches for local goods, while $\rho_i^b = 1$ means the buyer searches for foreign goods. When $\rho_i^b \in (0, 1)$, the buyer is indifferent between local and foreign produced goods and follows a mixed strategy where they search for foreign goods with probability ρ_i^b and for local goods with probability $1 - \rho_i^b$. We assume search abroad is costly. In particular, a buyer from i investing ρ_i incurs a utility cost $\Phi_i(\rho_i)$ to search in the foreign goods market. We assume that $\Phi_i' > 0, \Phi_i'' > 0$, $\Phi_i(0) = 0$ and $\Phi_i(1) = \infty$. This cost differs across regions, reflecting different regulations and technologies associated with searching abroad. In stationary equilibrium, these assumptions imply buyers are indifferent between searching abroad and at home.[10] Let $\widehat{\rho}_i \equiv \int_{b \in \mathbb{B}_i} \rho_i^b \, db$ denote the average search intensity for buyers in region i. In the following, we use ρ_i to denote ρ_i^b when no confusion may arise.

Matching. Following individual search decisions, buyers and sellers are matched pairwise in goods markets by an aggregate matching function. Since sellers are immobile, matches are formed in the seller's location j. Given $\widehat{\rho}_1$ and $\widehat{\rho}_2$, the total measure of matches in region j is given by $\mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j)$, which depends on the measures of active buyers and sellers in region j. The matching function is constant returns to scale, twice continuously differentiable, strictly increasing, strictly concave with respect to each argument and satisfies $\mathcal{M}_j(0, \mathcal{S}_j) = \mathcal{M}_j(\mathcal{B}_j, 0) = 0$ and $\mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j) \leq \min(\mathcal{B}_j, \mathcal{S}_j)$. In

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[10]While ρ_i is endogenous and determined by buyers' search effort (the intensive margin), the measure of buyers at the start of the DM (the extensive margin) is exogenous. In a closed economy, Lagos and Rocheteau (2005) keep the ratio of buyers to sellers fixed and introduce endogenous search intensity. Alternatively, Rocheteau and Wright (2005) have a fixed number of buyers and free entry by sellers while Rocheteau and Wright (2009) have a fixed total number of agents that can choose whether to be buyers or sellers. In either case, constant returns in matching implies a focus on market tightness rather than the overall size of the market.

region 1, the total measure of buyers and sellers are $\mathcal{B}_1 = (1 - \widehat{\rho}_1)n + \widehat{\rho}_2$ and $\mathcal{S}_1 = n$, respectively. Similarly in region 2, $\mathcal{B}_2 = \widehat{\rho}_1 n + (1 - \widehat{\rho}_2)(1 - \eta)$ and $\mathcal{S}_2 = 1$. The ratio of sellers to active buyers in region j, defined as market tightness, is $\vartheta_j \equiv \frac{\mathcal{S}_j}{\mathcal{B}_j}$. All matches are short-lived and destroyed at the end of the period.

Conditional on search decisions, an individual buyer's meeting probability is $\alpha_j(\vartheta_j) = \mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j)/\mathcal{B}_j = \mathcal{M}_j(1, \vartheta_j)$. The matching probability of a seller in region j is $\alpha_j(\vartheta_j)/\vartheta_j = \mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j)/\mathcal{S}_j = \mathcal{M}_j\left(\vartheta_j^{-1}, 1\right)$. The dependence of the matching probabilities on market tightness reflects the usual search and congestion externalities. We further assume $\alpha_j(0) = 0$, $\alpha_j'(0) \geq 0$, $\alpha_j(\infty) = 1$, and $\alpha_j'(\infty) = 0$. Table 1 summarizes buyers' meeting probability, $\alpha_j(\vartheta_j)$, across meeting types. Since matches form at random, $\alpha_j(\vartheta_j)/\vartheta_j$ is the matching probability of a seller in j. We denote the elasticity of the matching function in j as $\epsilon(\vartheta_j) \equiv 1 - \frac{\vartheta_j \, \alpha_j'(\vartheta_j)}{\alpha_j(\vartheta_j)}$.

Table 1: Buyers' Meeting Probabilities

	Seller from 1	Seller from 2
Buyer from 1	$(1 - \rho_1)\alpha_1(\vartheta_1)$	$\rho_1\alpha_2(\vartheta_2)$
Buyer from 2	$\rho_2\alpha_1(\vartheta_1)$	$(1 - \rho_2)\alpha_2(\vartheta_2)$

Preferences. The period utility of an active buyer in region j originally from i is given by

$$U^b(\rho_i, q_j, x, y) = -\Phi(\rho_i) + u(q_j) + x - y,$$

where ρ_i is the buyer's search intensity, q_j is consumption in the DM of region j, x is consumption of the numéraire, and y is production of the numéraire. We assume $u'(0) = \infty$, $u' > 0$ and $u'' < 0$ for $q_j > 0$. Similarly, the period utility of a seller in region j is given by

$$U^s(q_j, x, y) = -c(q_j) + x - y,$$

where $c(0) = c'(0) = 0$, $c' > 0$, and $c'' \geq 0$. We assume $c(q_j) = u(q_j)$ for some $q_j > 0$ and let q^* denote the solution to $c'(q^*) = u'(q^*)$.

Money. A single monetary authority issues a common currency for the union. The currency is intrinsically worthless, divisible, storable, and recognizable. The aggregate money supply in the CM of period t is M_t and the relative price of money in terms of the numéraire, ϕ_t, adjusts to clear the market. The gross growth rate of the money supply is constant over time and equal to $\gamma \equiv M_{t+1}/M_t \geq \beta$. New money is injected if $\gamma > 1$, or withdrawn if $\gamma < 1$, through lump sum

transfers or taxes to buyers at the beginning of the CM. The budget constraint for the currency union is therefore

$$\phi_t(M_{t+1} - M_t) = \mathcal{T}_t, \tag{1}$$

where $\mathcal{T}_t$ is the lump sum transfer (if $\gamma < 1$) or tax (if $\gamma > 1$) to buyers.

Timing. At the beginning of the DM, all buyers are in their exogenously assigned region of origin. A buyer from i chooses how much to invest to search for foreign-produced goods of region $j \neq i$. Conditional on this choice, buyers are then matched pairwise with sellers from j with probability α_j. After migrating and matching, the buyer is either in region 1 or 2, where terms of trade are determined through bilateral bargaining. At the start of the CM, buyers receive lump sum transfers of the common currency and adjust their portfolios.

3   Social Optimum

As a benchmark, we first consider the social planner's problem. The planner is constrained by the same frictions as private agents and chooses a stationary allocation, $\{(\rho_1, \rho_2), (q_1, q_2)\}$, to maximize total welfare for the union. Given market tightness $\vartheta_1 = \frac{n}{(1-\widehat{\rho}_1)n + \widehat{\rho}_2}$ and $\vartheta_2 = \frac{1}{(1-\widehat{\rho}_2) + \widehat{\rho}_1 n}$, steady state welfare is defined as the sum of agents' utilities in the two regions

$$\mathcal{W} \equiv n \frac{\alpha_1(\vartheta_1)}{\vartheta_1}[u(q_1) - c(q_1)] + \frac{\alpha_2(\vartheta_2)}{\vartheta_2}[u(q_2) - c(q_2)] - n\Phi_1(\widehat{\rho}_1) - \Phi_2(\widehat{\rho}_2), \tag{2}$$

where $n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}$ and $\frac{\alpha_2(\vartheta_2)}{\vartheta_2}$ are the measure of matches in region 1 and 2, respectively. Consequently, welfare for the union consists of the total trade surplus in DM of the two regions net of buyers' investment in search for foreign goods. The social planner's problem is

$$(q_1, q_2, \rho_1, \rho_2) \in \arg\max \mathcal{W} \tag{3}$$

subject to $\vartheta_1 = \frac{n}{(1-\widehat{\rho}_1)n + \widehat{\rho}_2}$ and $\vartheta_2 = \frac{1}{(1-\widehat{\rho}_2) + \widehat{\rho}_1 n}$.

Lemma 1. *The social optimum is given by $q_1 = q_2 = q^*$, $\rho_1 = \rho_1^*$ and $\rho_2 = \rho_2^*$ that solve*

$$u'(q^*) = c'(q^*), \tag{4}$$

$$\Phi_1'(\rho_1^*) = [\alpha_2(\vartheta_2^*)\epsilon(\vartheta_2^*) - \alpha_1(\vartheta_1^*)\epsilon(\vartheta_1^*)][u(q^*) - c(q^*)], \tag{5}$$

$$\Phi_2'(\rho_2^*) = [\alpha_1(\vartheta_1^*)\epsilon(\vartheta_1^*) - \alpha_2(\vartheta_2^*)\epsilon(\vartheta_2^*)][u(q^*) - c(q^*)], \tag{6}$$

where $\vartheta_1^* \equiv \frac{n}{(1-\rho_1^*)n+\rho_2^*}$ and $\vartheta_2^* \equiv \frac{1}{(1-\rho_2^*)+\rho_1^* n}$ is market tightness at the first best and $\epsilon(\vartheta_j^*) \equiv 1 - \frac{\vartheta_j^* \, \alpha_j'(\vartheta_j^*)}{\alpha_j(\vartheta_j^*)}$ is the elasticity of the matching function at the first best.

As is standard, (4) gives the efficient quantity of production per match by equating the marginal gain from consuming to the marginal cost of producing. From (5) and (6), the efficient number of trades requires the marginal cost of searching, $\Phi_1'(\rho_1^*)$ and $\Phi_2'(\rho_2^*)$, equals the difference in the social marginal contribution of search times the first-best surplus generated per trade.

4   Monetary Equilibrium

We now describe agents' decision problems in the CM and DM, respectively. We focus on stationary equilibrium where aggregate real balances are constant over time.

At the beginning of the CM, buyers choose consumption of the numéraire, labor, and real balances to bring forward next period. The buyer's state is his original location, indexed by $j = \{1, 2\}$, and their current holdings of real balances, $z_j \equiv \phi m_j \in \mathbb{R}_+$. Let W_j^b denote the buyer's value function in CM and V_j^b denote the buyer's value function in the ensuing DM. In what follows, variables with a prime denote next period's variables. The lifetime expected utility for a buyer from j is

$$W_j^b(z_j) = \max_{x,h,z_j' \geq 0} \{x - h + \beta \, V_j^b(z_j')\} \tag{7}$$

$$\text{s.t.} \quad x + \phi m_j' = h + z_j + T, \tag{8}$$

where z_j' is the buyer's portfolio of real balances taken into the next DM, $T \equiv \frac{\mathcal{T}}{1+n}$ is the per capita transfer of common currency (in units of the numéraire) and $V_j^b(z_j')$ is the buyer's continuation value in the next DM. Substituting $m_j' = z_j'/\phi$ from (8) into (7) yields

$$W_j^b(z) = z_j + T + \max_{z_j' \geq 0} \left\{ -\gamma z_j' + \beta \, V^b(z_j') \right\},$$

where γ is the gross growth rate of the money supply. Accordingly, the buyer's lifetime utility in the CM consists of his current period's real balances, the lump sum transfer, and his continuation value at the start of the next DM net of his investment in real balances. Hence, in order to hold z_j' units of real balances next period, the buyer must acquire $\gamma z_j'$ units of real balances in the current period. Since $W_j^b(z_j) = z_j + W_j^b(0)$, the buyer's CM value function is linear in his wealth, z_j. In addition, the buyer's choice of real balances next period is independent of his current period's real balances. So long as $\gamma \geq \beta$, sellers have no strict incentive to accumulate real balances in the DM. Consequently,

the CM value function of a seller with z_j is $W^s(z_j) = z_j + \beta V_j^s(0)$, which is also linear in z_j.

Following individual search decisions, terms of trade in the DM are determined by Kalai (1977) bargaining. In region j, a buyer acquires output q_j in exchange for payment d_j to the seller and receives a constant share θ_j of the total surplus, where $\theta_j \in (0,1]$ is the bargaining power of a buyer in j.[11] By the linearity of W_j^b, the surplus of a buyer who gets q_j in exchange for d_j is $u(q_j) + W_j^b(z_j - d_j) - W_j^b(z_j) = u(q_j) - d_j$, where the threat point is no trade. Similarly, the seller's surplus is $d_j - c(q_j)$. Terms of trade solve the bargaining problem

$$\max_{q_j, d_j} \{u(q_j) - d_j\} \text{ s.t. } u(q_j) - d_j = \frac{\theta_j}{1 - \theta_j}[d_j - c(q_j)] \text{ s.t. } d_j \leq z_j.$$

If $z_j \geq (1 - \theta_j)c(q^*) + \theta_j u(q^*)$, the buyer is unconstrained and the solution is $q = q^*$ and $d = (1 - \theta_j)c(q^*) + \theta_j u(q^*)$. Otherwise, $q_j < q^*$, and the buyer will just hand over all his real balances, $d_j = (1 - \theta_j)u(q_j) + \theta_j c(q_j)$. In that case, real balances are

$$z_1 = (1 - \theta_1)u(q_1) + \theta_1 c(q_1), \tag{9}$$

$$z_2 = (1 - \theta_2)u(q_2) + \theta_2 c(q_2). \tag{10}$$

It is important to highlight that q_1 and q_2 are the total quantity traded in bilateral meetings in regions 1 and 2, respectively. In general, buyers from the two regions can carry different portfolios and hence trade different quantities, but they will have the same total value. Hence, q_1 and q_2 are the total quantity traded by domestic and foreign buyers in regions 1 and 2.

Using the linearity of W_j^b, the lifetime value of a region 1 buyer in the DM is

$$V_1^b(z_1) = \max_{\rho_1 \in [0,1]} \{-\Phi_1(\rho_1) + (1 - \rho_1)\,\alpha_1(\vartheta_1)\,[u(q_1) - d_1] + \rho_1\,\alpha_2(\vartheta_2)\,[u(q_2) - d_2] + z_1 + W_1^b(0)\}.$$

A buyer in region 1 incurs an investment cost $\Phi_1(\rho_1)$ to search for goods produced in region 2. With probability $(1 - \rho_1)\alpha_1(\vartheta_1)$, the buyer meets a seller from 1, in which case he gets q_1 and transfers d_1 in exchange to the seller. With probability $\rho_1\alpha_2(\vartheta_2)$, the buyer searches for foreign-produced goods and upon meeting a foreign seller, receives q_2 in exchange for d_2. The term $z_1 + W_1^b(0)$ results from the linearity of the CM value function and is the value of proceeding to the next CM with one's portfolio intact.

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[11]Differences in bargaining power across regions can reflect different laws or market structures between regions.

Given the bargaining solution, the DM value function for a region 1 buyer is

$$V_1^b(z_1) = \max_{\rho_1 \in [0,1]} \{-\Phi_1(\rho_1) + (1-\rho_1)\,\alpha_1(\vartheta_1)\,\theta_1[u(q_1) - c(q_1)] + \rho_1\,\alpha_2(\vartheta_2)\,\theta_2[u(q_2) - c(q_2)] + z_1 + W_1^b(0)\}.$$

Similarly, the value function for a region 2 buyer is

$$V_2^b(z_2) = \max_{\rho_2 \in [0,1]} \{-\Phi_2(\rho_2) + \rho_2\,\alpha_1(\vartheta_1)\,\theta_1[u(q_1) - c(q_1)] + (1-\rho_2)\,\alpha_2(\vartheta_2)\,\theta_2[u(q_2) - c(q_2)] + z_2 + W_2^b(0)\}.$$

We now turn to buyers' search decisions at the beginning of the DM. When making this decision, individuals take as given market tightness and hence aggregate search rates. For a buyer in region 1, $\rho_1 \in [0,1]$ solves

$$\Phi_1'(\rho_1) = \alpha_2(\vartheta_2)\,\theta_2[u(q_2) - c(q_2)] - \alpha_1(\vartheta_1)\,\theta_1[u(q_1) - c(q_1)]. \tag{11}$$

Since $\Phi_1(\cdot)$ is strictly convex, the buyer's search intensity is uniquely defined and is continuous by the Theorem of the Maximum. The left side of (11) is the buyer's marginal cost of searching for foreign goods, $\Phi_1'(\rho_1)$, which must equal the marginal gain of search. A similar expression applies to a region 2 buyer, where $\rho_2 \in [0,1]$ solves

$$\Phi_2'(\rho_2) = \alpha_1(\vartheta_1)\,\theta_1[u(q_1) - c(q_1)] - \alpha_2(\vartheta_2)\,\theta_2[u(q_2) - c(q_2)]. \tag{12}$$

Conditions (11) and (12) equate the private, rather than social, cost and benefit of searching abroad. The dependence of buyers' matching probabilities on market tightness and the average search decisions of other buyers generates an externality typically not internalized in equilibrium. We revisit this efficiency issue later in the text.

We now describe the buyer's portfolio problem in the CM. Substituting $V_1^b(z_1)$ into $W_1^b(z)$, and using the linearity of W_1^b, the portfolio problem for a buyer in region 1 is given by

$$\max_{z_1 \in \mathbb{R}_+} \{-\iota z_1 - \Phi_1(\rho_1) + (1-\rho_1)\,\alpha_1(\vartheta_1)\,\theta_1[u(q_1) - c(q_1)] + \rho_1\,\alpha_2(\vartheta_2)\,\theta_2[u(q_2) - c(q_2)]\} \tag{13}$$

where $\iota \equiv (1+\rho)\gamma - 1 = \frac{\gamma - \beta}{\beta}$ can be interpreted as the nominal interest rate on an illiquid bond denominated in the common currency. Since (13) is continuous and maximizes over a compact set, a solution exists by the Theorem of the Maximum. The first order condition is

$$-\iota + (1-\rho_1)\,\alpha_1(\vartheta_1)\,\theta_1\,[u'(q_1) - c'(q_1)]\,\frac{\partial q_1}{\partial z_1} + \rho_1\,\alpha_2(\vartheta_2)\,\theta_2\,[u'(q_2) - c'(q_2)]\,\frac{\partial q_2}{\partial z_1} \leq 0, \tag{14}$$

where (14) holds at equality if $z_j > 0$. Kalai bargaining implies

$$\frac{\partial q_j}{\partial z_1} = \frac{1}{\theta_j c'(q_j) + (1 - \theta_j) u'(q_j)}.$$

As a result, $z_1 > 0$ solves

$$\iota = (1 - \rho_1)\, \alpha_1(\vartheta_1)\, \mathcal{L}_1(q_1) + \rho_1\, \alpha_2(\vartheta_2)\, \mathcal{L}_2(q_2), \tag{15}$$

where the term $\mathcal{L}_j(\cdot) \equiv \frac{\theta_j [u'(\cdot) - c'(\cdot)]}{\theta_j c'(\cdot) + (1-\theta_j) u'(\cdot)}$ is the marginal benefit a buyer receives from using the common currency to trade in the DM of region j. Similarly, $z_2 > 0$ solves

$$\iota = \rho_2\, \alpha_1(\vartheta_1)\, \mathcal{L}_1(q_1) + (1 - \rho_2)\, \alpha_2(\vartheta_2)\, \mathcal{L}_2(q_2). \tag{16}$$

Definition 1. *A stationary monetary equilibrium with endogenous search is a list $\{(z_1, z_2), (\rho_1, \rho_2), (q_1, q_2)\}$ that solves (9), (10), (11), (12), (15), (16), and market clearing in the money market, $z_1 + n z_2 = \phi M$.*

Equilibrium has a recursive structure. Once ρ_1 and ρ_2 are determined by (11) and (12), q_1 and q_2 are obtained from (15) and (16). Real balances are then pinned down by (9) and (10). We next compare the constrained efficient allocation given by (4), (5) and (6), with the equilibrium outcome. The following proposition describes the conditions under which an equilibrium is constrained efficient.

Proposition 1. *Equilibrium in the currency union achieves the social optimum if and only if*

$$\gamma = \beta, \tag{17}$$

$$\theta_1 = \epsilon(\vartheta_1), \tag{18}$$

$$\theta_2 = \epsilon(\vartheta_2), \tag{19}$$

where $\epsilon(\vartheta_j) = 1 - \frac{\vartheta_j\, \alpha'_j(\vartheta_j)}{\alpha_j(\vartheta_j)}$ is the elasticity of the matching function in region j.

According to Proposition 1, the monetary equilibrium coincides with the social optimum if and only if the Friedman rule holds at the union level and the Hosios condition holds at the regional level. Condition (17) is the Friedman rule, which ensures the efficient quantity of DM output per trade by contracting the money supply at the rate of time preference. This makes it costless to carry real balances across periods. While necessary, the Friedman rule is not sufficient for efficiency. Equations (18) and (19) are the corresponding Hosios conditions for each region, which ensures individual search

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decisions are socially optimal. Even when the Friedman rule holds, the Hosios condition is typically not satisfied, unless in the knife edge case when buyers' bargaining powers exactly equal to their contributions to the matching process as implied by (18) and (19). Thus, monetary equilibrium is generically inefficient – even at the Friedman rule – due to regional search externalities. Before introducing additional policy instruments, we next consider the positive implications of agents' search decisions.

5 One Sided Search

To highlight the main mechanisms of the model, we begin by studying one sided search where only region 1 buyers choose to search for foreign goods while search intensity in region 2 is fixed at $\bar{\rho}_2$. We first characterize properties of equilibrium with one sided search and then compare with the social optimum.

Proposition 2. *Given $\rho_2 = \bar{\rho}_2$, a unique steady state monetary equilibrium with one sided search exists and features $\rho_1 > 0$ if $\iota < \bar{\iota} \equiv \min\{\frac{\theta_1}{1-\theta_1}, \frac{\theta_2}{1-\theta_2}\}$ is small and*

$$\alpha_2 \left(\frac{1}{1-\bar{\rho}_2}\right) \theta_2 > \alpha_1 \left(\frac{n}{n+\bar{\rho}_2}\right) \theta_1. \tag{20}$$

In that case, comparative statics are given by Table 2.

Table 2: Comparative Statics with One Sided Search

	ι	θ_1	θ_2
ρ_1	$+$	$-$	$+$
q_1	$-$	$+$	$+$
q_2	$-$	$+$	$+$

With one sided search, there is a unique stationary monetary equilibrium with active search by region 1 households if (20) is satisfied and inflation is not too high. A necessary condition for (20) to hold is that trading conditions in region 2, captured by the matching probability and household bargaining power, are sufficiently large. From Table 2, higher inflation decreases trade in both regions and increases search for goods produced in region 2, if region 2 has more favorable terms of trade than region 1, i.e. θ_2 is large relative to θ_1. As expected, ρ_1 increases with θ_2, while the frequency of trades in region 2 decreases. In contrast, ρ_1 decreases with θ_1 since higher bargaining power with local producers makes purchasing local goods more attractive.

In general, equilibria may not be unique for all ι. Multiple steady states may arise due to a complementarity between output produced in the foreign region and households' search decisions.

From (15) and (16), output produced in region 1, q_1, is decreasing in the search rate, ρ_1, while output produced in region 2, q_2, is increasing in ρ_1. Intuitively, higher production in region 2 raises the expected surplus of searching for foreign goods by households in region 1, which increases their search intensity ρ_1 and hence the total number of trades in region 2, $\mathcal{B}_2\alpha_2(\vartheta_2) = [(1 - \bar{\rho}_2) + \rho_1 n]\alpha_2(\vartheta_2)$. This makes trade in region 2 more valuable which raises the value of money in region 2, z_2, and DM production, q_2.[12] In what follows, we assume the conditions in Proposition 2 are satisfied.

5.1 Efficiency

Achieving a constrained efficient allocation requires satisfying the conditions in Proposition 1. With one sided search, these imply production levels $q_1 = q_2 = q^*$, and search intensity for region 1 households ρ_1 that solves (11) given $\rho_2 = \bar{\rho}_2$. The next proposition summarizes conditions for underinvestment or overinvestment in search when the monetary authority implements the Friedman rule.

Proposition 3. *Suppose $\iota = 0$, and $\rho_2 = \bar{\rho}_2$ and (20) hold. If $\theta_1 > \epsilon(\vartheta_1)$, region 1 households underinvest in search for foreign goods, $\rho_1 < \rho_1^*$. If $\theta_1 < \epsilon(\vartheta_1)$, households overinvest in search, $\rho_1 > \rho_1^*$.*

The Friedman rule generates the efficient quantity per trade, $q_1 = q_2 = q^*$, but typically not the efficient total number of trades. From Proposition 1, individual search intensity ρ_1 is socially efficient if and only if $\theta_1 = \epsilon(\vartheta_1)$, i.e. the buyer's bargaining power equals their contribution to the matching process. In equilibrium, a household's incentive to search for foreign goods decreases with their bargaining power at home, i.e. $\partial\rho_1/\partial\theta_1 < 0$. Consider a small deviation from $\theta_1 = \epsilon(\vartheta_1)$. If θ_1 increases, i.e. $\theta_1 > \epsilon(\vartheta_1)$, the marginal gain from search given by the right side of (11) falls. As a result, ρ_1 decreases and households underinvest in search for foreign goods, $\rho_1 < \rho_1^*$. On the other hand, if θ_1 decreases, i.e. $\theta_1 < \epsilon(\vartheta_1)$, ρ_1 increases since now the marginal gain from search is higher. In this case, households overinvest in search, $\rho_1 > \rho_1^*$.

Numerical Examples

To illustrate additional properties of equilibrium, we consider numerical examples that demonstrate some of the model's positive implications when there is underinvestment or overinvestment in search. The matching function in region j is $\mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j) = \chi_j \frac{\mathcal{B}_j \mathcal{S}_j}{\mathcal{B}_j + \mathcal{S}_j}$, where $\chi_j > 0$ represents the efficiency of the matching process in region j. This implies buyers' matching probabilities are $\alpha_1(\vartheta_1) =$

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[12]The intuition for this multiplicity is similar to the complementarity between the value of money and agents' entry or search decisions in e.g. Rocheteau and Wright (2005) and Berentsen et al. (2007).

$\frac{\chi_1}{(1-\rho_1)+\bar{\rho}_2/n+1}$ and $\alpha_2(\vartheta_2) = \frac{\chi_2}{(1-\bar{\rho}_2)+\rho_1 n+1}$. DM utility and cost functions for production and search are respectively, $u(q_j) = \ln(q_j + b) - \ln(b)$ where $b > 0$, $c(q_j) = q_j$, and $\Phi_1(\rho_1) = \frac{\rho_1^2}{1-\rho_1}$.

Table 3 summarizes the parameter values used in the examples. We consider two values for household bargaining power in region 1, θ_1, which together with $\epsilon(\vartheta_1)$ determines whether there is under- or overinvestment in search. We set $\theta_2 = 0.55$ to ensure the condition for uniqueness, (20), is satisfied under both parameterizations. The annual discount rate is set to $\rho = 3\%$, which gives $\beta = 0.97$. As a benchmark, we set $\gamma = \beta$, which is the Friedman rule, but later consider examples with higher values for γ.[13]

Table 3: Parameter Values

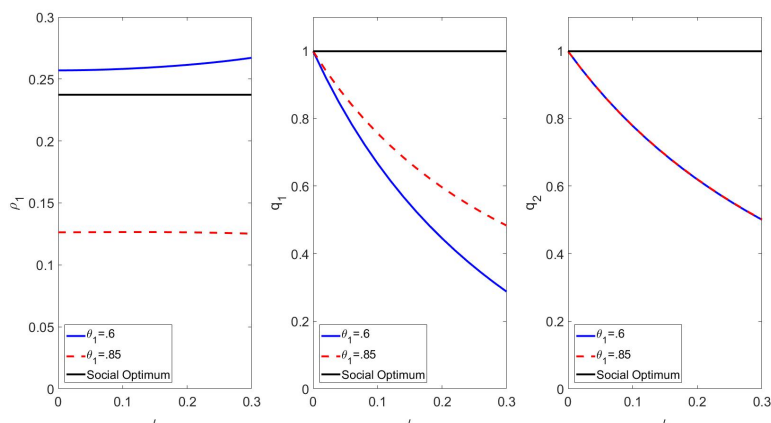| β	b	n	χ_1	χ_2	θ_1	θ_2	$\bar{\rho}_2$	γ
0.97	0.001	0.03	1.17	1	0.6, 0.85	0.55	0.0385	0.97

When $\theta_1 = 0.85$, the matching elasticity in region 1 is $\epsilon(\vartheta_1) = 0.41$. Since $\theta_1 > \epsilon(\vartheta_1)$, households underinvest in search, i.e. the equilibrium search rate $\rho_1 = 0.13$ is below the first best $\rho_1^* = 0.24$. In contrast, when $\theta_1 = 0.6$, $\epsilon(\vartheta_1) = 0.64$, which implies overinvestment in search, i.e. $\rho_1 = 0.26$.

Figure 1 illustrates the effects of money growth on search intensity for foreign goods, ρ_1, and regional production, q_1 and q_2, for the two parameterizations of θ_1. As γ increases, DM output in both regions fall while search intensity for foreign goods increase. In addition, the percent change in DM output is larger when households face a lower bargaining power in region 1. In our example, the percent change in the search rate in region 1 is smaller compared to DM production in region 1, which suggests the intensive margin (output per trade) responds more to monetary policy than the extensive margin (total number of trades).

While higher money growth increases search, this effect is only second order while the negative intensive margin effect is first order. Hence the Friedman rule is still optimal. While the Friedman rule delivers efficiency in DM production in both regions, search is inefficiently high when $\theta_1 = 0.6$ and inefficiently low when $\theta_1 = 0.85$. Under positive nominal interest rates, union wide welfare is always higher when buyer's bargaining power in region 1 is larger. This finding suggests the drop in DM production is larger when buyer's in region 1 face a lower bargaining power, relative to the increase in

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[13]The choice of the underlying parameters try to capture some features of the Belgium economy. More precisely, Region 1 represents Belgium and region 2 is the EMU. During 2000 to 2014, the average population size of Belgium relative to the rest of the EMU is $n = 0.03$ (OECD). To get matching efficiencies, χ_1 and χ_2, ideally we would like time series data on bilateral migration flows at the EMU level. To get matching efficiencies, χ_1 and χ_2, we normalize $\chi_2 = 1$ and set χ_1 to the relative labor efficiency of Belgium to the rest of the EMU. In 2012, Labor productivity, measured as GDP per hour worked, between Belgium and the EMU was $\chi_1/\chi_2 = 1.17$, which gives $\chi_1 = 1.17$. to determine $\bar{\rho}_2$, we use information regarding the total contribution of DM production to GDP from EMU visitors to Belgium. According to the World Travel and Tourism Council (WTTC) over the period 1995 until 2007, the average contribution was 6.4%. Moreover, 60% of all tourists that visited Belgium came from the EMU. Thus an approximate measure of the total contribution of tourism to GDP from EMU visitors is then 3.85%.

Figure 1: Effects of Money Growth on Search and Production with One Sided Search

the search rate. Moreover, as γ increases, the welfare difference between the two economies increases. We next consider how fiscal instruments may be able to correct this extensive margin distortion.

5.2 Fiscal Policy

From the previous section, efficiency on all margins requires satisfying the Friedman rule at the union level and Hosios conditions at the regional level. Without active search, monetary policy is sufficient, and the Friedman rule achieves the first best. With endogenous search, buyers do not internalize the externalities their search decisions have on matching probabilities in the goods market. Since monetary policy alone is not enough to correct this distortion, we consider an additional instrument that can affect search intensity at the regional level. Here we consider fiscal policy through region specific tax or subsidies in the DM.

In the following, we consider a proportional tax or subsidy on DM production. With one sided search, the scheme only applies to producers in region 2 (we consider the two sided case where producers from both regions are subsidized or taxed in Section 6). To implement this policy, we assume the government has access to a costless record keeping technology that keeps track of the identity and production of producers but not identify of households in the DM.[14] For instance, the government can record DM production since producers are in fixed and known locations due to immobile factors of production.[15] Hence while the fiscal authority cannot directly tax household search decisions, they can indirectly affect their search intensity by taxing DM profits of producers,

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[14]If all agents are anonymous in the DM, the government cannot directly tax productive activities. However, when the identity of *some agents* are known and a record of their production is available to the government, the fiscal authority can tax or subsidize DM activity.

[15]The fact that producers' identities are known does not preclude money being socially useful since all households are still anonymous. In addition, anonymity of some private agents does not preclude the government from raising tax revenues. As in Chari and Kehoe (1993), taxes directly levied on firms are feasible as their output is observable.

which affects the expected surplus of search for foreign goods.

As in Gomis-Porqueras and Peralta-Alva (2010), the government institutes a proportional subsidy or tax on DM goods implemented through lump sum monetary injections or withdrawals in the beginning of the CM. After CM trade, but before the next DM, the monetary authority implements changes in the money supply through (a different) lump sum transfer to households. Implicit in our set up is that the currency union is also a fiscal union: the fiscal authority is taxing (subsidizing) region 2 production but finances it by subsidizing (taxing) both regions through the common inflation tax.[16] As a result, there is redistribution between households and producers in region 2.[17] The resulting government constraints are then

$$\phi \mathcal{S} = \tau_2 \, \alpha_2(\vartheta_2) \left[\rho_1 + (1 - \bar{\rho}_2) \right] z_2, \tag{21}$$

$$\phi M(\gamma - 1) = \phi \mathcal{S} + \mathcal{T}, \tag{22}$$

where (21) defines the size of the subsidy paid to sellers in region 2 and (22) is the government budget constraint. Finally, through market clearing,

$$\phi M = z_1 + n z_2. \tag{23}$$

It is important to highlight the subsidy is purely monetary as is financed through money printing. However, the lump sum tax that agents face in the CM is both monetary and fiscal, whenever the money supply shrinks.[18] Notice also that when $\tau_2 \neq 0$, there is a redistribution of resources to producers in the DM.

Terms of trade in region 2 are still determined through Kalai bargaining, but now includes the subsidy/tax, $\tau_2 \in [-1, 1]$, on region 2 production. Terms of trade now solve

$$\max_{q_2, d_2} \left\{ u(q_2) - c(q_2) \right\}$$

$$\text{s.t. } u(q_2) - d_2 = \frac{\theta_2}{1 - \theta_2} [d_2(1 + \tau_2) - q_2]$$

$$d_2 \leq z_2,$$

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[16]This view is consistent with a classic proposal from Kenen (1969) on the importance of fiscal integration in a monetary union. See also the literature on fiscal policy in monetary unions, e.g. Beetsma and Uhlig (1999), Cooper and Kempf (2004), Chari and Kehoe (2008), and references therein.

[17]This feature plays an important role in internalizing the congestion externality arising from search decisions.

[18]In most of the models that are based on Lagos and Wright (2005) that consider Friedman rule, the CM lump sum tax is as much monetary than fiscal as is needed to collect taxes to finance the return on fiat money.

where q_2 also depends implicitly on ρ_1. If $z_2 > \frac{(1-\theta_2)u(q^*)+\theta_2 c(q^*)}{1+\tau_2\theta_2}$, the household has enough real balances to purchase the efficient quantity q^*. Otherwise, the household is cash constrained and will just hand over all his real balances to the seller so that $d_2 = z_2$ to obtain q_2. The payment in region 2 is given by

$$d_2 = \frac{(1-\theta_2)u(q_2)+\theta_2 c(q_2)}{1+\tau_2\theta_2}.$$

When $d_2 = z_2$, there are two cases. When $\tau_2 \in (0, 1]$, the fiscal authority enacts a subsidy and

$$\frac{\partial q_2}{\partial \tau_2} = \frac{z_2\theta_2}{(1-\theta_2)u'(q_2)+\theta_2 c'(q_2)} > 0.$$

Hence output produced in region 2 is increasing in the subsidy, $\tau_2 \in (0, 1]$. In addition, notice τ_2 also affects buyers' effective bargaining power, $\tau_2\theta_2$, which can make fiscal policy especially useful.[19] In particular, $\tau_2\theta_2$ affects the magnitude of the effective buyer's surplus in region 2 relative to region 1. As we saw from Proposition 3, this bargaining power affects the search decision by buyers in region 1. Note that a larger subsidy in region 2 increases local output, q_2, and the search intensity ρ_1. This situation allows the possibility for both monetary and fiscal policies to increase welfare when search is inefficiently high or low.

When the fiscal authority enacts a proportional tax, i.e. $\tau_2 \in [-1, 0)$, DM output decreases with τ_2 ($\frac{\partial q_2}{\partial \tau_2} < 0$) and the search rate decreases ($\frac{\partial \rho_1}{\partial \tau_2} < 0$). In a later part of the paper we show how this fiscal scheme can raise welfare whenever there is underinvestment in search activities.
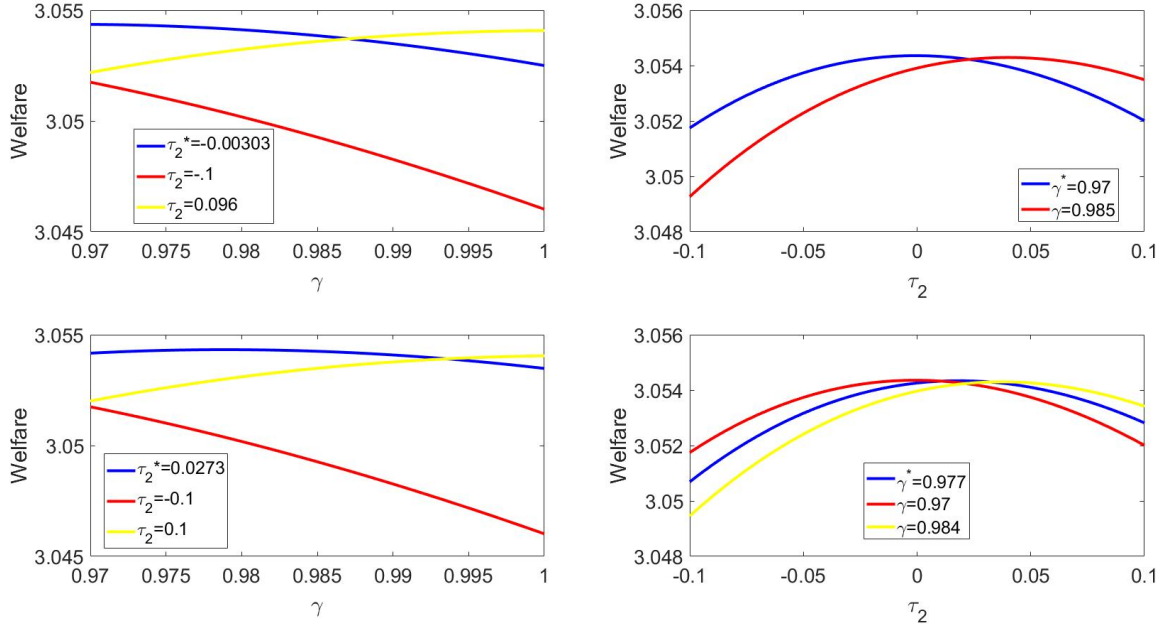
5.3 Optimal Monetary and Fiscal Policies

We now consider the design of optimal policy following the Ramsey tradition. In this context, the government chooses the monetary-fiscal policy mix to maximize union wide welfare, taking as given the government budget constraint and equilibrium decisions of private agents. The available instruments are the money growth rate for the union, γ, and the production tax/subsidy on region 2's DM production, τ_2. Importantly, the monetary and fiscal authorities set (γ, τ_2) once and for all and can commit to their policies. The policy problem is given by

$$\max_{\gamma, \tau_2} \left\{ n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}\ [u(q_1)-c(q_1)] - n\Phi_1(\rho_1) + \frac{\alpha_2(\vartheta_2)}{\vartheta_2}\ [u(q_2)-c(q_2)] - \Phi_2(\rho_2) \right\} \tag{24}$$

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[19]Instead of a proportional scheme, the government could alternatively propose a lump sum subsidy on region 2 sellers. In that case, there would be less change on production since a lump sum subsidy would only affect the total surplus.

Figure 2: Welfare and Optimal Policies (Top: Overinvestment, Bottom: Underinvestment)

subject to (11) and

$$\iota = (1 - \rho_1)\alpha_1(\vartheta_1)\mathcal{L}_1(q_1) + \rho_1\alpha_2(\vartheta_2)(1 + \tau_2\theta_2)\mathcal{L}_2(q_2), \tag{25}$$
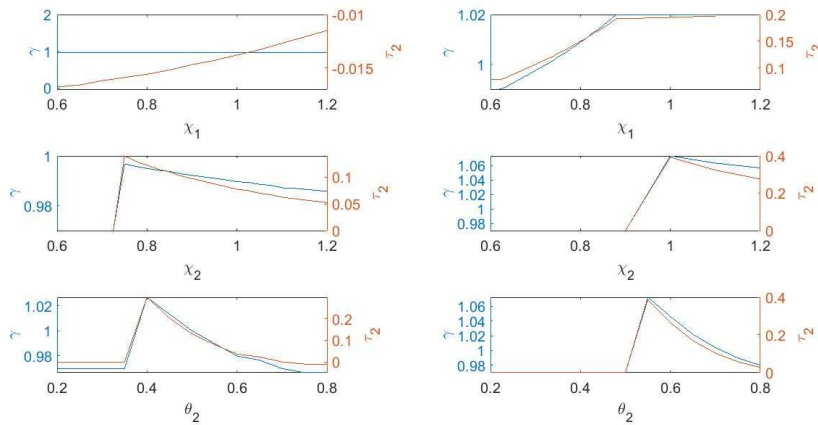
$$\iota = \rho_2\alpha_1(\vartheta_1)\mathcal{L}_1(q_1) + (1 - \rho)2)\alpha_2(\vartheta_2)(1 + \tau_2\theta_2)\mathcal{L}_2(q_2). \tag{26}$$

The first three constraints correspond to equilibrium decisions of private agents: (11) determines search intensity for foreign goods, while (25) and (26) determine q_1 and q_2, i.e. production in the two regions given taxes/subsidies in region 2.

Since the optimal policy mix depends on whether households under- or overinvest in search, we consider two cases: $\theta_1 = 0.6$, which implies overinvestment in search at the Friedman rule, and $\theta_1 = 0.85$, which implies underinvestment. Figure 2 shows the optimal policy mix, (γ, τ_2) when $\theta_1 = 0.6$ (top panel) and $\theta_1 = 0.85$ (bottom panel). The blue lines in the left panel plot union welfare against the money growth rate γ, assuming the fiscal authority follows the optimal policy τ_2^*. The red (yellow) line assumes a value of τ below (above) the optimal τ_2^*. Similarly, the right panel plot welfare against the tax rate τ_2.

When households overinvest in search ($\theta_1 = 0.6$), optimal monetary policy is the Friedman rule. In contrast, with underinvestment in search ($\theta_1 = 0.85$), optimal monetary policy is a money growth rate above the Friedman rule, $\gamma = 0.977$. This is illustrated by the top left panel for $\theta_1 = 0.6$ and the

Figure 3: Comparative Statics for Optimal Policies (Left: Overinvestment, Right: Underinvestment)

bottom left panel for $\theta_1 = 0.85$. In both cases, there is deflation, $\gamma < 1$, and hence all buyers face lump sum taxes in the CM. These results highlight the importance of having a monetary-fiscal union that implements the union wide inflation tax. Since the deviation from the Friedman rule is not large in this example, our findings suggests that the intertemporal distortion may be more relevant than the congestion externality induced by endogenous search (this is fairly robust across parameterizations). Indeed, the top left panel shows the Friedman rule can still be optimal even when other instruments are available to the government. This arises when there is a production tax in the DM, e.g. when $\tau_2 = -0.1$. In contrast, the Friedman rule is not optimal when there is a production subsidy, e.g. $\tau_2 = 0.096$ when $\theta_1 = 0.6$. These results highlight the importance of the redistribution from buyers to sellers of region 2 to correct the congestion externality.

We now study how these optimal policies respond with changes in model parameters, i.e. matching efficiency in both regions and the buyer's bargaining power in region 2. Figure 3 summarizes comparative statics with respect to the parameters governing optimal monetary policy γ (when fiscal policy is fixed at its optimal value) and optimal fiscal policy τ_2 (assuming monetary policy is fixed at its optimal value) when $\theta_1 = 0.6$ (left) and $\theta_1 = 0.85$ (right).

The blue lines denote the optimal money growth rate, γ, while the orange lines denote the optimal tax/subsidy rate in region 2, τ_2. As we can see from From Figure 3, higher matching efficiency and household bargaining power in region 2 result in smaller fiscal responses. These are more accentuated when buyers are under-investing in their search decisions.

6 Two Sided Search

We now allow households from both regions to search for foreign-produced goods. In this case, ρ_1 and ρ_2 jointly solve the first order conditions associated with the buyers' search decisions which are given by equations (11) and (12). Other than regional differences in bargaining powers and matching efficiencies, regions also differ in terms of the search costs. In what follows we consider economies where $\Phi_1(\rho_1) = \frac{\rho_1^2}{1-\rho_1}$ and $\Phi_2(\rho_2) = \Phi_1(\rho_2) - a\rho_2$, where a is a small positive constant.

We first consider equilibrium where monetary policy follows the Friedman rule, $\iota = 0$, and there is no fiscal policy. As a result, changes in bargaining power and matching efficiency do not produce any distortion along the intensive margin, i.e. on q_1 and q_2. However, the extensive margin, ρ_1 and ρ_2, can still be affected by the congestions externality and be not equal to the efficient search intensity. The effects of changes in the economic environment on search decisions are summarized by the following table (see the Appendix for derivations).

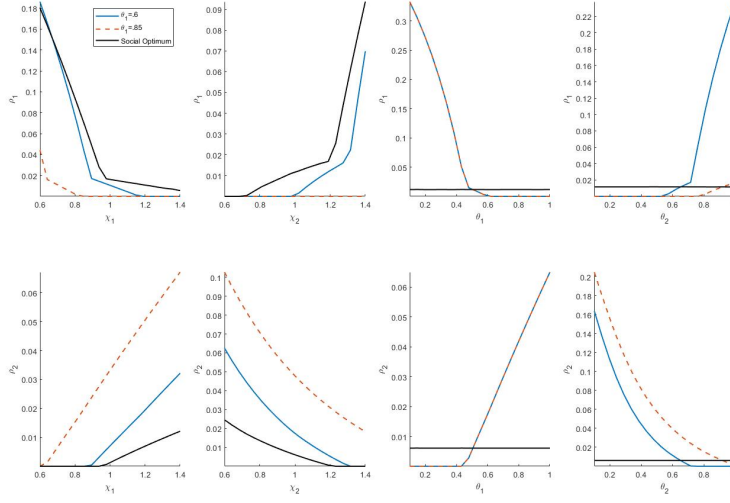Table 4: Comparative Statics with Two Sided Search

	θ_1	θ_2	χ_1	χ_2
ρ_1	$-$	$+$	$-$	$+$
ρ_2	$+$	$-$	$+$	$-$

These comparative statics are consistent with those found in the one sided search case. Specifically, an increase in household bargaining power or matching efficiency at home decreases search for foreign goods. As in the one sided case, buyers choose to relocate to regions they expect to have larger surpluses. These are associated with regions where households have larger bargaining power and matching efficiencies.

Similarly, the responsiveness of search intensity in both regions depends on how far equilibrium search decisions are from the first best. To illustrate this effect, Figure 4 shows how the search rate ρ_1 and ρ_2 respond to changes in structural parameters. In particular, we analyze changes in χ_1, χ_2, θ_1, and θ_2. The red and blue lines in the figure show two different parameterizations. The blue lines corresponds to an economy with $\theta_1 = 0.6$, which delivers overinvestment in search activities. The red line represents an economy with $\theta_1 = 0.85$, which results in underinvestment. Finally, the black line shows the Hosios parameterization, which deliver constrained-efficient search rates. Throughout the rest of the paper, we consider the same benchmark parametrization as the previous section.

From Figure 4, equilibrium search intensity in region 2 is smaller than search intensity in region 1. Moreover, the effects of bargaining power and matching efficiency are monotonic and piecewise linear. These depend on whether equilibrium search rates are above or below the corresponding first

Figure 4: Comparative Statics with Two Sided Search (Red: Underinvestment; Blue: Overinvestment)

best search rates. Relative to the first best, search intensity in both regions are more responsive to changes in bargaining power than matching efficiency. This is the case as the bargaining power directly effects the expected surplus. Changes in matching efficiency deliver smaller departures from the first best. This indicates that the associated changes in the extensive margin are relatively small. Moreover, when buyers from both regions can search, the smaller region is more responsive to changes in the matching efficiency.

6.1 Optimal Monetary and Fiscal Policies

With two sided search, there is an additional instrument available to the fiscal authority that can mitigate inefficiencies from search decisions in region 2. Now the optimal policy mix consists of a money growth rate for the union and production tax/subsidies in both regions (γ, τ_1, τ_2) that maximize social welfare of the union subject to the optimal decisions by private agents and feasibility constraints. Formally, the policy problem with two sided search is given by

$$\max_{\gamma, \tau_1, \tau_2} \left\{ n \frac{\alpha_1(\vartheta_1)}{\vartheta_1} \left[u(q_1) - c(q_1) \right] - n\Phi_1(\rho_1) + \frac{\alpha_2(\vartheta_2)}{\vartheta_2} \left[u(q_2) - c(q_2) \right] - \Phi_2(\rho_2) \right\} \tag{27}$$

subject to (11), (12) and

$$\iota = (1 - \rho_1)\alpha_1(\vartheta_1)(1 + \tau_1\theta_1)\mathcal{L}_1(q_1) + \rho_1\alpha_2(\vartheta_2)(1 + \tau_2\theta_2)\mathcal{L}_2(q_2), \tag{28}$$

$$\iota = \rho_2\alpha_1(\vartheta_1)(1 + \tau_1\theta_1)\mathcal{L}_1(q_1) + (1 - \rho)2)\alpha_2(\vartheta_2)(1 + \tau_2\theta_2)\mathcal{L}_2(q_2), \tag{29}$$

the subsidy size, government budget constraint, and market clearing condition, respectively:

$$\phi\mathcal{S} = \tau_1 \, \alpha_1(\vartheta_1) \, [(1 - \rho_1)n + \rho_2]z_1 + \tau_2\alpha_2(\vartheta_2)[\rho_1 + (1 - \rho_2)]z_2, \tag{30}$$

$$\phi M(\gamma - 1) = \phi\mathcal{S} + \mathcal{T}, \tag{31}$$

$$\phi M = z_1 + nz_2. \tag{32}$$

The first four constraints correspond to equilibrium decisions of private agents. In particular, equations (11) and (12) determine the optimal search for households in region 1 an d 2 respectively, while (28) and (29) determine production in the two regions given taxes/subsidies in region 1 and 2. In addition, (30) defines the size of the subsidy paid to producers in region 2, (31) represents the government budget constraint, and (32) is the money market clearing condition. Since the central bank interacts with all private agents, i.e. both households and sellers, in the CM through lump sum monetary transfers, there is a redistribution of resources to producers when $\tau_2 \neq 0$.

Using the parametrization in our previous numerical example, The resulting optimal policy delivers the Friedman rule and zero DM production taxes/subsidies in both regions. These results suggests that correcting the intensive margin is more important than having the extensive margin closer to the efficient level. The qualitative results obtained in the two sided search case are in line with the results under one sided search. In both cases, the authorities in the monetary union prioritize the intensive margin rather than the extensive one.

7   Conclusion

In this paper, we constructed an open economy model of a currency union with endogenous search and studied how monetary and fiscal policies can correct distortions along the intensive and extensive margins. Due to regional spillovers from agents' search decisions, equilibrium is generically inefficient. While monetary policy can eliminate the intensive margin distortion by running the Friedman rule, search rates can still be too high or too low, unless in the knife edge case when the Hosios condition is also satisfied. To correct this inefficiency, we introduce fiscal policy that can tax or subsidize production at the regional level.

Key to our analysis is the assumption that the monetary union is also a fiscal union: the fiscal authority is taxing (subsidizing) production but finances it by subsidizing (taxing) both regions through the common inflation tax. Consequently, fiscal policy leads to redistribution between households and producers and hence can be set to minimize the externality that arises when households search for

foreign goods. When only households from one region choose to search, the optimal policy mix entails a deviation from the Friedman rule. However, either a production subsidy, if there is underinvestment in search activities, or a production tax, if there is overinvestment, can be optimal. In contrast, when households from both regions can search, our numerical example shows that the optimal policy is the Friedman rule and zero taxes in both regions. Together our findings dramatize how the (intensive margin) distortions in quantity traded can be more socially beneficial to correct than the (extensive margin) distortions arising from search decisions. A potential avenue for future work is to allow for further heterogeneity and examine its consequences for trade-off between inflation and search.

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Appendix

Proof of Lemma 1

To obtain the social optimum, we differentiate the social welfare function (2) with respect to q_1, q_2, ρ_1, and ρ_2. The first order condition with respect to q_j yields $u'(q_j) = c'(q_j)$ and hence $q_1 = q_2 = q^*$. The first order conditions with respect to $\rho_j > 0$ are

$$\left\{ n \left(\frac{\alpha_1'(\vartheta_1)}{\vartheta_1} - \frac{\alpha_1(\vartheta_1)}{\vartheta_1^2} \right) \frac{\partial \vartheta_1}{\partial \rho_1} + \left(\frac{\alpha_2'(\vartheta_2)}{\vartheta_2} - \frac{\alpha_2(\vartheta_2)}{\vartheta_2^2} \right) \frac{\partial \vartheta_2}{\partial \rho_1} \right\} [u(q^*) - c(q^*)] = n\Phi_1'(\rho_1)$$

$$\left\{ \left(\frac{\alpha_1'(\vartheta_1)}{\vartheta_1} - \frac{\alpha_1(\vartheta_1)}{\vartheta_1^2} \right) \frac{\partial \vartheta_1}{\partial \rho_2} + \left(\frac{\alpha_2'(\vartheta_2)}{\vartheta_2} - \frac{\alpha_2(\vartheta_2)}{\vartheta_2^2} \right) \frac{\partial \vartheta_2}{\partial \rho_2} \right\} [u(q^*) - c(q^*)] = \Phi_2'(\rho_2)$$

where

$$\frac{\partial \vartheta_1}{\partial \rho_1} = \vartheta_1^2 \; ; \; \frac{\partial \vartheta_2}{\partial \rho_2} = \vartheta_2^2$$

$$\frac{\partial \vartheta_1}{\partial \rho_2} = -\frac{\vartheta_1^2}{n} \; ; \; \frac{\partial \vartheta_2}{\partial \rho_1} = -n\vartheta_2^2.$$

Upon substituting and rewriting, we obtain

$$\Phi_1'(\rho_1^*) = \left[\alpha_2(\vartheta_2^*) \left(1 - \frac{\alpha_2'(\vartheta_2^*)\vartheta_2^*}{\alpha_2(\vartheta_2^*)} \right) - \alpha_1(\vartheta_1^*) \left(1 - \frac{\alpha_1'(\vartheta_1^*)\vartheta_1^*}{\alpha_1(\vartheta_1^*)} \right) \right] [u(q^*) - c(q^*)],$$

$$\Phi_2'(\rho_2^*) = \left[\alpha_1(\vartheta_1^*) \left(1 - \frac{\alpha_1'(\vartheta_1^*)\vartheta_1^*}{\alpha_1(\vartheta_1^*)} \right) - \alpha_2(\vartheta_2^*) \left(1 - \frac{\alpha_2'(\vartheta_2^*)\vartheta_2^*}{\alpha_2(\vartheta_2^*)} \right) \right] [u(q^*) - c(q^*)],$$

which are (5) and (6) in the main text. $\square$

Proof of Proposition 1

To show the Friedman rule, $\iota = 0$, produces the efficient quantity of trade, $q_1 = q_2 = q^*$, consider the equilibrium values for q_1 and q_2 given by (15) and (16). Setting $\iota = 0$ gives $u'(q_j) = c'(q_j)$ and hence, $q_1 = q_2 = q^*$. Equilibrium search rates are given by (11) and (12) at $\iota = 0$, or

$$\Phi_1'(\rho_1) = [\alpha_2(\vartheta_2)\theta_2 - \alpha_1(\vartheta_1)\theta_1] [u(q^*) - c(q^*)], \tag{33}$$

$$\Phi_2'(\rho_2) = [\alpha_1(\vartheta_1)\theta_1 - \alpha_2(\vartheta_2)\theta_2] [u(q^*) - c(q^*)]. \tag{34}$$

It is now easy to see the equilibrium search rates equal the first best allocations if (33) and (34) coincide with (5) and (6), respectively. This is happen if and only if $\theta_j = \epsilon(\vartheta_j)$. $\square$

Proof of Proposition 2

Equilibrium has a recursive structure. Once ρ_1 is determined by 41, we obtain $q_1(\rho_1)$ and $q_2(\rho_1)$ from (39) and (40). Real balances z_1 and z_2 are then pinned down by (9) and (10). Since $\Phi_1(0) = \Phi_1'(0) = 0$ and $\Phi_1'' > 0$, the left side of (41) is increasing in ρ_1. To show the equilibrium exists and is unique, we first establish there is a unique solution, z_1, rewritten as $\max_{z_1 \geq 0} O(z_1; \iota)$ where

$$O(z_1; \iota) \equiv -\iota z_1 - \Phi(\rho_1) + (1 - \rho_1)\, \alpha_1(\vartheta_1)\, \theta_1[u(q_1) - c(q_1)] + \rho_1\, \alpha_2(\vartheta_2)\, \theta_2[u(q_2) - c(q_2)]$$

With no loss in generality, we can restrict the choice for z_1 to the compact interval $[0, z_1^*]$ where $z_1^* \equiv (1 - \theta_1)c(q^*) + \theta_1 u(q^*)$. If $z_1 > z_1^*$, then $O'(z_1; \iota) = -\iota$. Moreover, $O(z_1; \iota)$ is continuous in z_1. Hence a solution exists by the Theorem of the Maximum and $\max_{z_1 \geq 0} O(z_1; \iota)$ is continuous in ι. A similar argument applies to the solution to z_2. Under Kalai bargaining, money is valued if and only if $\iota < \min\{\frac{\theta_1}{1-\theta_1}, \frac{\theta_2}{1-\theta_2}\}$.

Next, we establish there is a unique solution, ρ_1 to (15). Since $\Phi(\cdot)$ is strictly convex, ρ_1 is uniquely defined and continuous by the Theorem of the Maximum. Hence a solution exists. To make sure there exist a unique positive solution for ρ_1 at $\iota = 0$, we need $\Phi_1(0) > 0$ which is satisfied if (20) holds.

Comparative statics for ρ_1 are obtained by applying the implicit function theorem and Cramer's rule:

$$\frac{\partial \rho_1}{\partial \theta_1} = \frac{\alpha_1(\vartheta_1^*)[u(q^*) - c(q^*)]}{\frac{\partial F_1}{\partial \rho_1}\big|_{\iota=0}} < 0,$$

$$\frac{\partial \rho_1}{\partial \theta_2} = -\frac{\alpha_2(\vartheta_1^*)[u(q^*) - c(q^*)]}{\frac{\partial F_1}{\partial \rho_1}\big|_{\iota=0}} > 0,$$

where

$$\frac{\partial F_1}{\partial \rho_1}\bigg|_{\iota=0} = -\Phi_1''(\rho_1^*) + [\alpha_2'(\vartheta_2^*)(-n\vartheta_2^{*2})\theta_2 - \alpha_1'(\vartheta_1^*)\vartheta_1^{*2}\theta_1][u(q^*) - c(q^*)] < 0. \tag{35}$$

If ι is small and (20) holds, $\partial \rho_1/\partial \theta_1 < 0$ and $\partial \rho_1/\partial \theta_2 > 0$. At $\iota = 0$, comparative statics with respect to ι are

$$\frac{d\rho_1}{d\iota}\bigg|_{\iota=0} = 0$$

$$\frac{dq_1}{d\iota}\bigg|_{\iota=0} = \frac{1}{\alpha_1(\vartheta_1^*)\mathcal{L}_1'(q^*)} < 0 \tag{36}$$

$$\frac{dq_2}{d\iota}\bigg|_{\iota=0} = \frac{1}{\alpha_2(\vartheta_2^*)\mathcal{L}_2'(q^*)} < 0$$

where we used that

$$\mathcal{L}_j'(q^*) = \frac{\theta_j(c'u'' - u'c'')}{[\theta_j c' + (1 - \theta_j)u']^2}\bigg|_{q=q^*} < 0. \tag{37}$$

□

Proof of Proposition 3

We measure steady state welfare for the union at the start of DM, before households make their search decisions and portfolio choices.

$$\mathcal{W} \equiv n \frac{\alpha_1(\vartheta_1)}{\vartheta_1}[u(q_1) - c(q_1)] + \frac{\alpha_2(\vartheta_2)}{\vartheta_2}[u(q_2) - c(q_2)] - n\Phi_1(\widehat{\rho}_1) - \Phi_2(\widehat{\rho}_2), \tag{38}$$

where $n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}$ and $\frac{\alpha_2(\vartheta_2)}{\vartheta_2}$ are the measure of matches in DM of regions 1 and 2. Equilibrium values for z_1, z_2, ρ_1, q_1, and q_2 are given by (9), (10), (11), (12), (15), and (16).

At $\iota = 0$, $q_1 = q_2 = q^*$. Social welfare is

$$\mathcal{W}^{FR} \equiv n \frac{\alpha_1(\vartheta_1^{FR})}{\vartheta_1^{FR}}[u(q^*) - c(q^*)] + \frac{\alpha_2(\vartheta_2^{FR})}{\vartheta_2^{FR}}[u(q^*) - c(q^*)] - n\Phi_1(\rho_1^{FR}) - \Phi_2(\rho_2^{FR}),$$

where ϑ_1^{FR} is market tightness at $\iota = 0$ and ρ_1^{FR} solves (33). Defining $\epsilon(\vartheta_1) \equiv 1 - \frac{\vartheta_1 \, \alpha_1'(\vartheta_1)}{\alpha_1(\vartheta_1)}$ as the elasticity of the matching function in region 1, the socially optimal search rates solves (5). We next establish $\partial \rho_1/\partial \theta_1 < 0$ at $\iota = 0$:

$$\left.\frac{\partial \rho_1}{\partial \theta_1}\right|_{\iota=0} = \frac{-\alpha_1(\vartheta_1)[u(q^*) - c(q^*)]}{\Phi''(\rho_1) + \left[\theta_2\alpha_2'(\vartheta_2)\vartheta_2^2/n - \theta_1\alpha_1'(\vartheta_1)\vartheta_1^2\right][u(q^*) - c(q^*)]} < 0,$$

where we assumed the condition for uniqueness, (20), holds. If $\theta_1 = \epsilon(\vartheta_1)$, $\rho_1 = \rho_1^*$ from Proposition 1. Now consider a small deviation from $\theta_1 = \epsilon(\vartheta_1)$. If θ_1 increases, i.e. $\theta_1 > \epsilon(\vartheta_1)$, ρ_1 falls since $\partial \rho_1/\partial \theta_1 < 0$. As a result, $\rho_1 < \rho_1^*$. Similarly, if θ_1 decreases, i.e. $\theta_1 < \epsilon(\vartheta_1)$, ρ_1 now increases. Hence, $\rho_1 > \rho_1^*$. $\square$

Comparative Statics Derivations for Table 4

In these derivations, we assume $\Phi_2(\rho_2) = \Phi_1(\rho_1) - a(\rho_2 + b\rho_2^2)$. At the Friedman rule, $q_1 = q_2 = q^*$. Letting $S(q^*) \equiv u(q^*) - c(q^*)$, the first order conditions for households' search decisions solve

$$\Phi_1'(\rho_1) = S(q^*)[\alpha_2\theta_2 - \alpha_1\theta_1],$$

$$\Phi_2'(\rho_2) = S(q^*)[\alpha_1\theta_1 - \alpha_2\theta_2],$$

where $\alpha_1 = \frac{\chi_1}{(1-\rho_1)+\rho_2/n+1}$ and $\alpha_2 = \frac{\chi_2}{(1-\rho_2)+\rho_1 n+1}$. This immediately implies $\Phi_1(\rho_1) = \frac{\rho_1^2}{1-\rho_1}$, $\Phi_1'(\rho_1) = \frac{2\rho_1}{1-\rho_1} - \left(\frac{\rho_1}{1-\rho_1}\right)^2 > 0$, and $\Phi_1''(\rho_1) = 2(1-\rho_1)^{-3} > 0$. Similarly, $\Phi_2(\rho_2) = \Phi_1(\rho_1) - a(\rho_2 + b\rho_2^2)$, $\Phi_2'(\rho_1) = -a(1 + 2b\rho_2)$, $\Phi_2''(\rho_2) = -2ab$. We therefore need $a > 0$ and $b \leq 0$.

The comparative statics with respect to θ_1 and θ_2 are

$$
\begin{bmatrix} \frac{\partial \rho_1}{\partial \theta_1} \\ \frac{\partial \rho_2}{\partial \theta_2} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_1}{\partial \rho_1} & \frac{\partial F_1}{\partial \rho_2} \\ \frac{\partial F_2}{\partial \rho_1} & \frac{\partial F_2}{\partial \rho_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial \theta_1} \\ \frac{\partial F_2}{\partial \theta_2} \end{bmatrix}
$$

where $F_1 \equiv -\Phi_1'(\rho_1) + S(q^*)[\alpha_2\theta_2 - \alpha_1\theta_1] = 0$ and $F_2 \equiv -\Phi_2'(\rho_2) + S(q^*)[\alpha_1\theta_1 - \alpha_2\theta_2] = 0$, and

$$
\frac{\partial F_1}{\partial \rho_1} = \Phi_1''(\rho_1) + S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_1} - \theta_1\frac{\partial \alpha_1}{\partial \rho_1}\right] = \Phi_1''(\rho_1) + S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_1} - \theta_1\frac{\partial \alpha_1}{\partial \rho_1}\right],
$$

$$
\frac{\partial F_1}{\partial \rho_2} = -\frac{\partial \Phi_1'(\rho_1)}{\partial \rho_2} + S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_2} - \theta_1\frac{\partial \alpha_1}{\partial \rho_2}\right] = -S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_2} - \theta_1\frac{\partial \alpha_1}{\partial \rho_2}\right],
$$

$$
\frac{\partial F_2}{\partial \rho_1} = -\frac{\partial \Phi_2'(\rho_2)}{\partial \rho_1} + S(q^*)\left[\theta_1\frac{\partial \alpha_1}{\partial \rho_1} - \theta_2\frac{\partial \alpha_2}{\partial \rho_1}\right] = -S(q^*)\left[\theta_1\frac{\partial \alpha_1}{\partial \rho_1} - \theta_2\frac{\partial \alpha_2}{\partial \rho_1}\right],
$$

$$
\frac{\partial F_1}{\partial \rho_1} = -\Phi_2''(\rho_2) + S(q^*)\left[\theta_1\frac{\partial \alpha_1}{\partial \rho_2} - \theta_2\frac{\partial \alpha_2}{\partial \rho_2}\right] = -\Phi_2''(\rho_2) + S(q^*)\left[\theta_1\frac{\partial \alpha_1}{\partial \rho_2} - \theta_2\frac{\partial \alpha_2}{\partial \rho_2}\right].
$$

$$
\begin{bmatrix} \frac{\partial F_1}{\partial \rho_1} & \frac{\partial F_1}{\partial \rho_2} \\ \frac{\partial F_2}{\partial \rho_2} & \frac{\partial F_2}{\partial \rho_2} \end{bmatrix}^{-1} = - \begin{bmatrix} \frac{\partial F_1}{\partial \rho_1}\frac{\partial F_2}{\partial \rho_2} & -\frac{\partial F_1}{\partial \rho_2}\frac{\partial F_2}{\partial \rho_1} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_2}{\partial \rho_2} & -\frac{\partial F_1}{\partial \rho_2} \\ -\frac{\partial F_2}{\partial \rho_1} & \frac{\partial F_1}{\partial \theta_1} \end{bmatrix}
$$

where we define

$$
DD^{-1} \equiv \begin{bmatrix} \frac{\partial F_1}{\partial \rho_1}\frac{\partial F_2}{\partial \rho_2} & -\frac{\partial F_1}{\partial \rho_2}\frac{\partial F_2}{\partial \rho_1} \end{bmatrix}^{-1}.
$$

Therefore,

$$
DD = \left[\Phi_1''(\rho_1) + S(q^*)\right]\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_1} - \theta_1\frac{\partial \alpha_1}{\partial \rho_1}\right]\left[-\Phi_2''(\rho_2) + S(q^*)\right]\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_2} - \theta_1\frac{\partial \alpha_1}{\partial \rho_2}\right]
$$

$$
-S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_2} - \theta_1\frac{\partial \alpha_1}{\partial \rho_2}\right]S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_1} - \theta_1\frac{\partial \alpha_1}{\partial \rho_1}\right]
$$

$$
= \left[\Phi_1'' - \Phi_2''\right] - S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_1} - \theta_1\frac{\partial \alpha_1}{\partial \rho_1}\right]\Phi_2''
$$

$$
-S(q^*)\left[\theta_2\frac{\partial \alpha_2}{\partial \rho_2} - \theta_1\frac{\partial \alpha_1}{\partial \rho_2}\right]\Phi_1''.
$$

This implies $DD > 0$. It also follows that

$$
\frac{\partial F_1}{\partial \theta_1} = -S(q^*)\alpha_1,
$$

$$
\frac{\partial F_2}{\partial \theta_1} = S(q^*)\alpha_1.
$$

Upon substitution, we therefore get $\frac{\partial \rho_1}{\partial \theta_1} < 0$ and $\frac{\partial \rho_2}{\partial \theta_1} > 0$.

Similar calculations confirm that $\frac{\partial \rho_1}{\partial \theta_2} > 0$, $\frac{\partial \rho_2}{\partial \theta_2} < 0$, $\frac{\partial \rho_1}{\partial \chi_1} < 0$, $\frac{\partial \rho_2}{\partial \theta_1} > 0$, $\frac{\partial \rho_1}{\partial \chi_2} > 0$, $\frac{\partial \rho_2}{\partial \chi_2} < 0$. Notice for all these results to hold, we need $b < 0$ (so that $\Phi_2(\rho_2)$ has some curvature). If e.g. $b = 0$, then ρ_1 is not affected by changes in $(\theta_1, \theta_2, \chi_1, \chi_2)$.

$\square$

Equilibrium Conditions Under Functional Forms in Sections 5 and 6

Given the functional forms in Section 5, $q_1(\rho_1)$ and $q_2(\rho_1)$ are given by

$$q_1(\rho_1) = \frac{\theta_1 - (1 - \theta_1)F_1(\rho_1)}{\theta_1(1 + F_1(\rho_1))} - b, \tag{39}$$

$$q_2(\rho_1) = \frac{\theta_2 - (1 - \theta_2)F_2(\rho_1)}{\theta_2(1 + F_2(\rho_1))} - b, \tag{40}$$

where

$$F_1(\rho_1) \equiv \frac{\iota(1 - \rho_1 - \bar{\rho}_2)}{(1 - \bar{\rho}_2)(1 - \rho_1) - \rho_1 \bar{\rho}_2} \frac{(1 - \rho_1) + \bar{\rho}_2/n + 1}{\chi_1},$$

$$F_2(\rho_1) \equiv \frac{\iota(1 - \rho_1 - \bar{\rho}_2)}{(1 - \bar{\rho}_2)(1 - \rho_1) - \rho_1 \bar{\rho}_2} \frac{(1 - \bar{\rho}_2) + \rho_1 n + 1}{\chi_2}.$$

The search rate, ρ_1, solves

$$\frac{\rho_1(2 - \rho_1)}{(1 - \rho_1)^2} = \frac{\chi_2 \theta_2 [\ln(q_2(\rho_1) + b) - \ln(b) - q_2(\rho_1)]}{(1 - \bar{\rho}_2) + \rho_1 n + 1} - \frac{\chi_1 \theta_1 [\ln(q_1(\rho_1) + b) - \ln(b) - q_1(\rho_1)]}{(1 - \rho_1) + \bar{\rho}_2/n + 1}. \tag{41}$$

Given the functional forms from Section 6, resulting equilibrium search rates ρ_1 and ρ_2 solve

$$\frac{\rho_1(2 - \rho_1)}{(1 - \rho_1)^2} = \frac{\chi_2 \theta_2 [\ln(q_2(\rho_1, \rho_2) + b) - \ln(b) - q_2(\rho_1, \rho_2)]}{(1 - \rho_2) + \rho_1 n + 1} - \frac{\chi_1 \theta_1 [\ln(q_1(\rho_1, \rho_2) + b) - \ln(b) - q_1(\rho_1, \rho_2)]}{(1 - \rho_1) + \rho_2/n + 1}, \tag{42}$$

$$\frac{\rho_2(2 - \rho_1)}{(1 - \rho_2)^2} = \frac{\chi_1 \theta_1 [\ln(q_1(\rho_1, \rho_2) + b) - \ln(b) - q_1(\rho_1, \rho_2)]}{(1 - \rho_1) + \rho_2/n + 1} - \frac{\chi_2 \theta_1 [\ln(q_2(\rho_1, \rho_2) + b) - \ln(b) - q_2(\rho_1, \rho_2)]}{(1 - \rho_2) + \rho_1 n + 1}; \tag{43}$$

where $q_1(\rho_1)$ and $q_2(\rho_1)$ are given by

$$q_1(\rho_1, \rho_2) = \frac{\theta_1 - (1 - \theta_1)F_1(\rho_1, \rho_2)}{\theta_1(1 + F_1(\rho_1, \rho_2))} - b \quad , \quad q_2(\rho_1, \rho_2) = \frac{\theta_2 - (1 - \theta_2)F_2(\rho_1, \rho_2)}{\theta_2(1 + F_2(\rho_1, \rho_2))} - b,$$

with

$$F_1(\rho_1, \rho_2) \equiv \frac{\iota(1 - \rho_1 - \rho_2)}{(1 - \rho_2)(1 - \rho_1) - \rho_1 \bar{\rho}_2} \frac{(1 - \rho_1) + \rho_2/n + 1}{\chi_1};$$

$$F_2(\rho_1, \rho_2) \equiv \frac{\iota(1 - \rho_1 - \rho_2)}{(1 - \rho_2)(1 - \rho_1) - \rho_1 \rho_2} \frac{(1 - \rho_2) + \rho_1 n + 1}{\chi_2}.$$